CSCI 8535 Multi Robot Systems

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Talk Outline

- Recap on Course Introduction, Syllabus, etc.
- Announcements
- Graph Theory – fundamentals
- Rendezvous problem

Course Introduction

This is primarily a **research oriented, seminar-style** course covering the topics of control, communication, cooperation, and coordination aspects in multi-robot systems.

It enables students to understand, devise, and solve problems in multi-robot systems and the course will have project-based assignments.

Course Outline

General topics to be covered:

- Multi-robot Rendezvous and Formation Control
- Multi-agent Cooperation and Coordination
- Security and adversarial actions
- Applications of Multi-Robot Systems

Goals of the Course

Graduate-only course.

- Give you a good intuition of ***Multi Robot Systems (MRS)*** modeling and control
  - The essential theoretical tools for MRS
  - How to implement and simulate MRS
  - How to solve real-world multi-robots problems

- You will be able to work on a MRS projects

- After the course, you will:
  - Know the essential theoretical tools for MRS
  - Know how to implement and simulate MRS
  - Know how to solve real-world problems
  - Develop and present a research project
  - Learn something about mobile robots

Requisites of the Course

Requirements:

- (Hard) Programming background and skills (Python or C++)
- (Soft) Working knowledge of simulation tools (Matlab or V-REP or ROS Gazebo, etc.)
- (Hard) Rudimentary mathematical analysis
- (Hard) Linear algebra
- (Soft) Some control theory
- (Soft) Graph theory fundamentals
- (Soft) Probability theory fundamentals

Course Style

- Seminar-style lectures
  - Each student will be assigned a paper to read and present it to the class (as if it’s their own work)
  - Each student will need to critically and constructively review the papers not assigned to them

- Project-based practical assignments and exam

Grading Criteria

In-class participation and Attendance: 10%

Assignments/Paper Reviews: 20%

Paper Presentations: 20%

Mini Project (Midterm): 20% (Project assigned by the Instructor)

Research Project (Final Project): 30% (Project chosen by the student in teams)

Office hours

Office hours of the instructor:

Tuesday and Thursday 2 -3 pm.

Location: Boyd Room 519/520.

If this schedule does not work for you, then send me an email to set up an appointment.

Announcements

First paper assignment for review (by all students):

Presenter: Instructor

Paper: "Consensus Control of Distributed Robots Using Direction of A rrival of Wireless Signals" R. Parasuraman and BC. Min, Intl. Sym. On Distributed Autonomous Robotic Systems (DARS), 2018.

Paper accessible here.
http://cobweb.cs.uga.edu/~ramviyas/pdf/DARS2018_Parasuraman.pdf

Also, uploaded to *eLC* and *Slack.*

Graph Theory, Rendezvous Problem, and Formation Control Problem

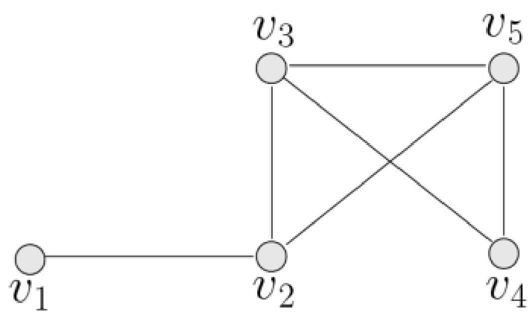
Courtesy of slides from Dr. Andrej Pronobis, U. Washington and KTH Sweden

https://www.pronobis.pro/teaching/mas/

Graph Theory

- Great tool for analyzing networks

Undirected Graphs



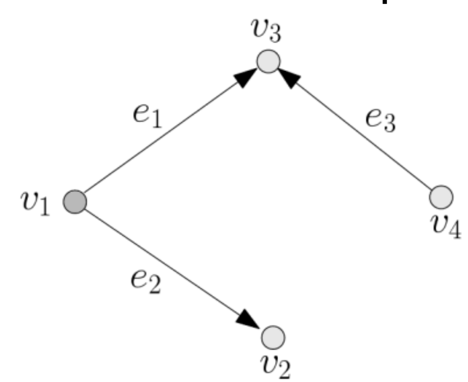
$\mathcal{G} = (V, E)$

Set of vertices

Set of edges

Unordered pair $\{v_i, v_j\} \in E \subseteq [V]^2$

Directed Graphs



$\mathcal{D} = (V, E)$

Ordered pair $(v_i, v_j) \in E$

Tail

Head

- Neighborhood of a vertex $N(i) = \{v_j \in V \mid v_i v_j \in E\}$

Slides courtesy of Dr. Andrej Pronobis

Algebraic Graph Theory

- Adjacency matrix

$$[A(\mathcal{G})]_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E, \\ 0 & \text{otherwise.} \end{cases} \qquad [A(\mathcal{D})]_{ij} = \begin{cases} 1 & \text{if } (v_j, v_i) \in E(\mathcal{D}) \\ 0 & \text{otherwise,} \end{cases}$$

- Degree matrix (undirected graph)

Degree of vertex $d(v_i)$ represents cardinality of neighborhood set $N(i)$

$$\Delta(\mathcal{G}) = \begin{pmatrix} d(v_1) & 0 & \cdots & 0 \\ 0 & d(v_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d(v_n) \end{pmatrix}$$

- In-degree matrix (directed graph)

$d(v_i)$ represents the in-degree (counts incoming edges only)

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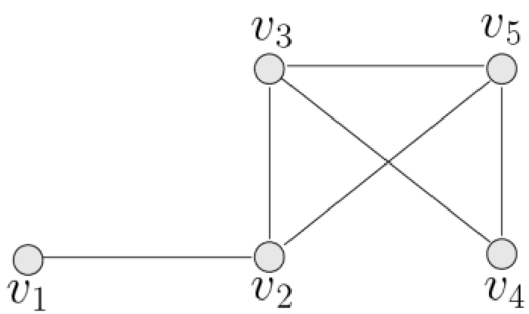
Graph Laplacian

- For undirected graphs

$$L(\mathcal{G}) = \Delta(\mathcal{G}) - A(\mathcal{G})$$



$$L(\mathcal{G}) = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{pmatrix}$$

- For directed graphs – In-degree Laplacian

$$L(\mathcal{D}) = \Delta(\mathcal{D}) - A(\mathcal{D})$$

Slides courtesy of Dr. Andrej Pronobis

Properties of Laplacian

- Symmetric and positive semi-definite
- Eigenvalues can be ordered as

$$\lambda_1(\mathcal{G}) \leq \lambda_2(\mathcal{G}) \leq \cdots \leq \lambda_n(\mathcal{G})$$

- Smallest eigenvalue is always zero

$$\lambda_1(\mathcal{G}) = 0$$

- Is the graph connected?
  - If for every pair of vertices there is a path
  - IFF $\lambda_2(\mathcal{G}) > 0$
  - As many connected sub-graphs as zero eigenvalues

Slides courtesy of Dr. Andrej Pronobis

Types of Graphs

A graph is said to be connected when there is at least one path (connection/link) from every vertex to every other vertex.

- Path graph or a line graph: each node is connected only once (without a loop)
- Cycle graph: graph structure looks like a cycle (loop)
- Complete graph: all vertices (nodes) are connected to all other vertices
- Tree graph: a connected graph without any cycles

Quick references for Graph Theory

Nice internet resource available in the internet for learning graph theory notations and definitions:

http://www.personal.kent.edu/~rmuhamma/GraphTheory/MyGraphTheory/defEx.htm

Rendezvous Problem – Agreement

- Agents agree on a value of a parameter
- Definition
  - n dynamic agents
  - Interconnected via relative links
  - Agent's state depends on the sum of its relative states w.r.t. a subset of other agents
- Applications
  - Distributed estimation in sensor networks
  - Flocking/swarming

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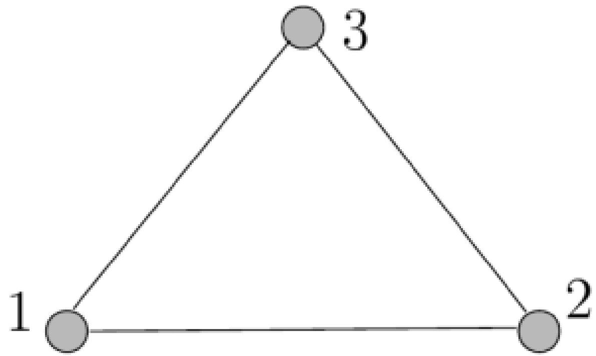
Rendezvous Problem

- Rendezvous problem
  - Agent’s state is its location – mobile robot
  - Agents should meet at one point in space
- Example: agreement protocol over a triangle

$$\dot{x}_3 = (x_1 - x_3) + (x_2 - x_3)$$



$$\dot{x}_1 = (x_2 - x_1) + (x_3 - x_1) \qquad \dot{x}_2 = (x_1 - x_2) + (x_3 - x_2)$$

- Links pull robots towards each other

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State-space Representation

- Continuous-time state-space model

$$\dot{x}_i(t) = \sum_{j \in N(i)} (x_j(t) - x_i(t)), \quad i = 1, \ldots, n$$

$$\boxed{\dot{x}(t) = -L(\mathcal{G})\, x(t)}$$

- For directed graphs: use in-degree Laplacian

$$\boxed{\dot{x}(t) = -L(\mathcal{D})x(t)}$$

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State-space Models in Matlab

- Dynamics of a system specified using continuous time-invariant state-space model

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- Create state-space model

```
sys = ss(A,B,C,D)
```

- In our case:

$$\dot{x}(t) = -L(\mathcal{G})\,x(t) \qquad \dot{x}(t) = -L(\mathcal{D})x(t)$$

A

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Simulating

- Simulate
  - Initial condition response

```
[y,t,x] = initial(sys,x0,t)
```

$$\dot{\mathbf{x}}(t) = A\mathbf{x} \qquad \mathbf{y}(t) = C\mathbf{x}$$

  - Response to arbitrary inputs

```
[y,t,x] = lsim(sys,u,t,x0)
```

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \qquad \mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- Basic toolkit for defining and visualizing problems available in course materials

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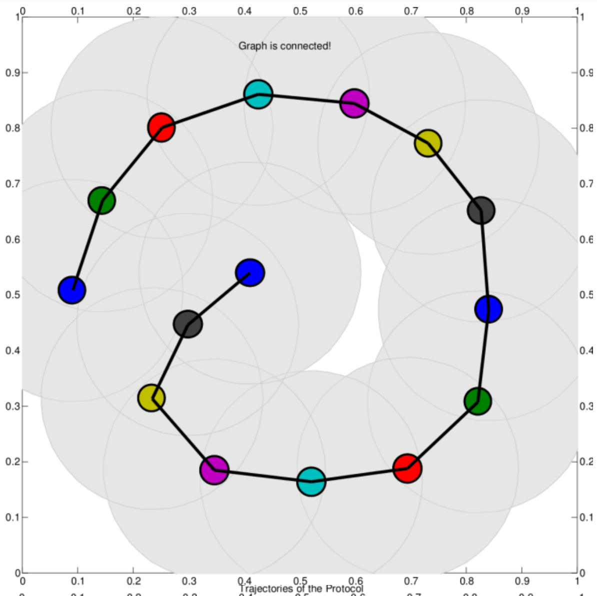
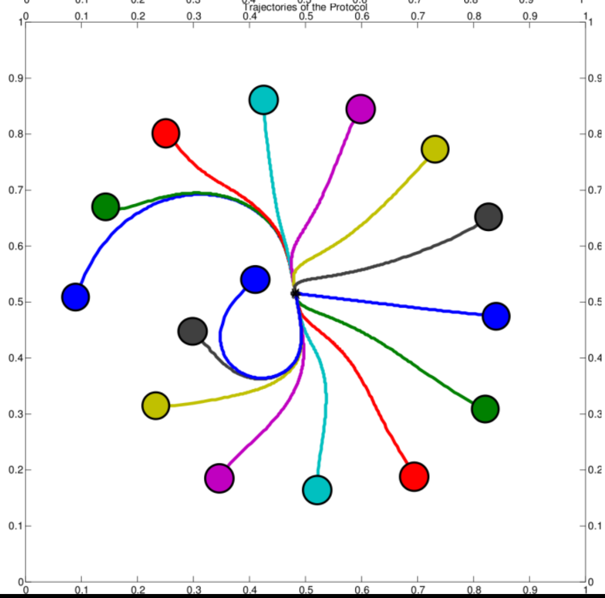
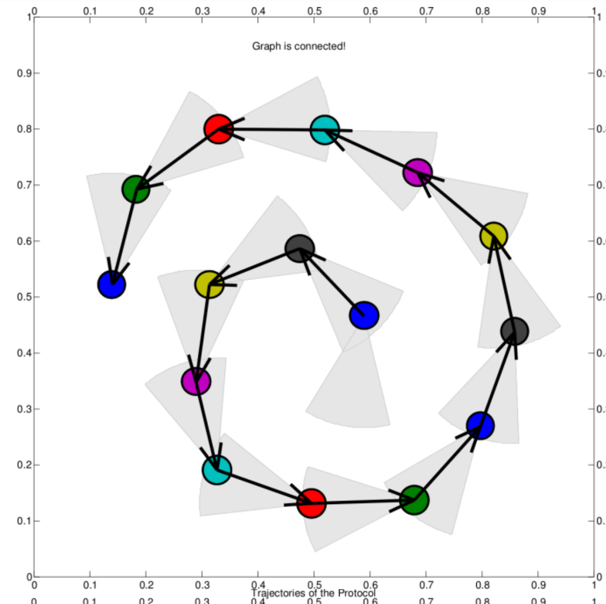
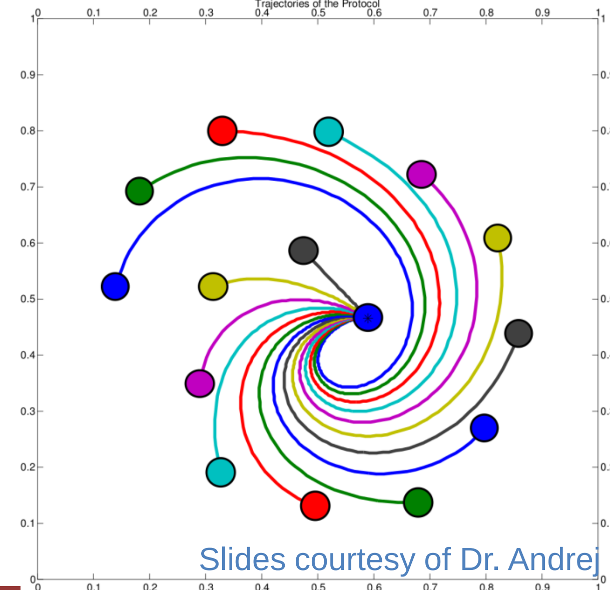
Simulating Trajectories





Run in Matlab





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Formation Control Problem

- Mobile agents move in order to realize a geometrical pattern (formation)
  - Appear often in biological systems (e.g. geese)
- Formations can be specified in several ways
  - **Shape**
    - Specified in terms of points

$$\Xi = \{\xi_1, \ldots, \xi_n\},\ \xi_i \in \mathbf{R}^p,\ i = 1, \ldots, n,$$

    - Translationally invariant

$$x_i = \xi_i + \tau$$

Slides courtesy of Dr. Andrej Pronobis

State-space Representation

- Let's define τ_i as displacement from target

$$\tau_i(t) = x_i(t) - \xi_i,\ i = 1, \ldots, n$$

- Now, apply the agreement protocol to τ_i

$$\dot{\tau}_i(t) = - \sum_{j \in N_f(i)} (\tau_i(t) - \tau_j(t))$$

- Since $\dot{\tau}_i(t) = \dot{x}_i(t)$, $\tau_i(t) - \tau_j(t) = x_i(t) - x_j(t) - (\xi_i - \xi_j)$

$$\dot{x}_i(t) = - \sum_{j \in N_f(i)} (x_i(t) - x_j(t)) - (\xi_i - \xi_j)$$

$$\dot{x}(t) = -L(\mathcal{G})\, x(t) + L(\mathcal{G})\, \Xi$$

Analogous for directed graphs

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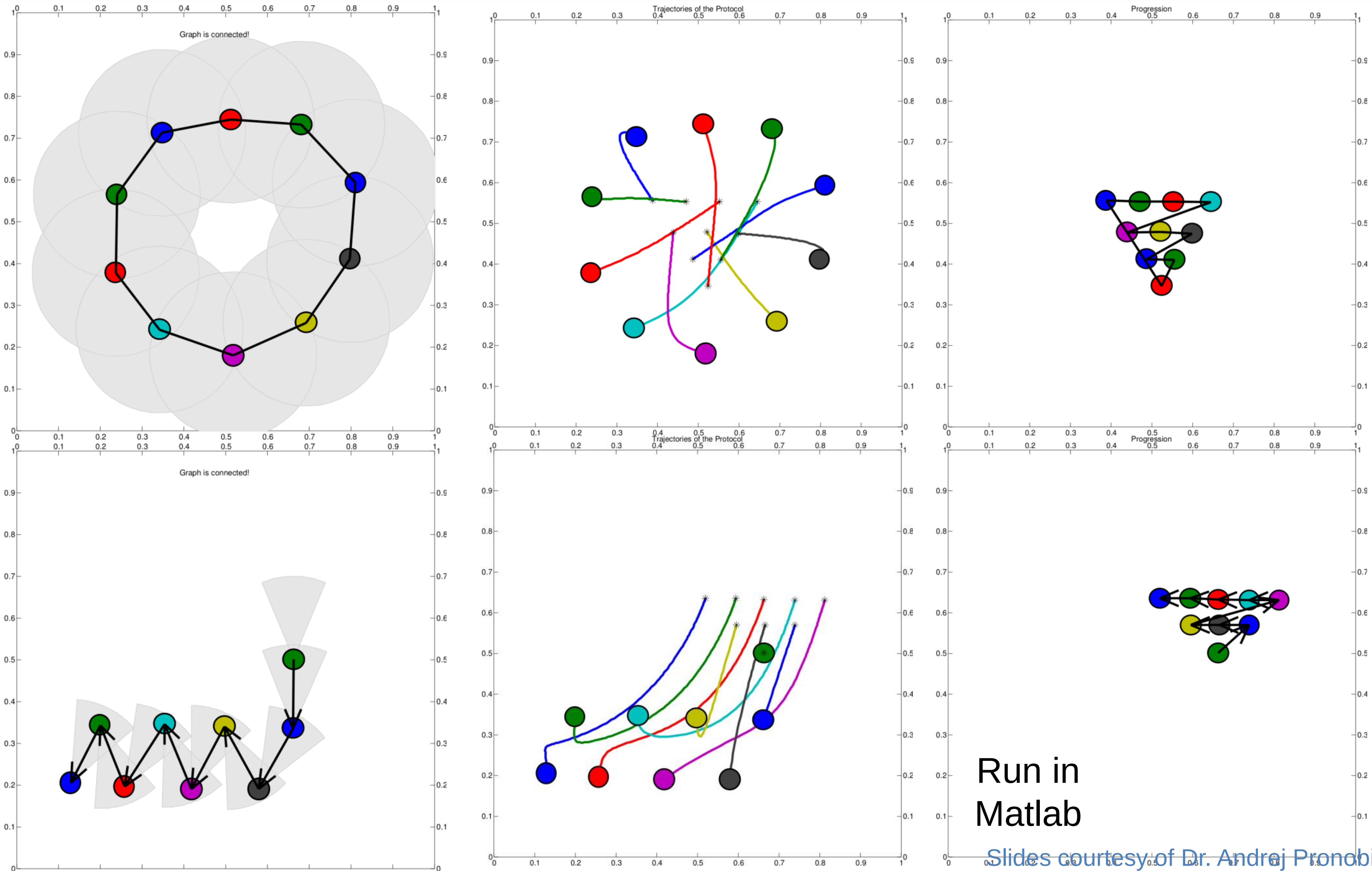
Simulating Trajectories



Run in Matlab

Slides courtesy of Dr. Andrej Pronobis

Summary

- Intuition about what multi-agent and multi-robot systems are and how to solve control problems
  - From theory to simulations
- Multiple applications in robotics
  - When no global maps and central coordination
- What’s next?
  - Dynamic and random networks
  - Switching between formations and control problems
  - Networks as systems (with inputs & outputs)
- Try this at home!

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