Homework Assignment 1

CSCI 2670 Introduction to Theory of Computing, Fall 2018

Due Thursday August 30, 2018

1. Let $x$ and $y$ be two positive integers. Prove (by construction) that if $y - x \geq 2$, there is an integer $z$ between $x$ and $y$, i.e., $x < z < y$.

2. CSCI 2670 class has 48 students with the average score 79 in their final exam. Prove (by contradiction) that there is at least one student with final exam score $\geq 79$.

3. Let alphabet $\Sigma = \{0, 1\}$. $\Sigma^0 = \{\epsilon\}$, where $\epsilon$ is the special empty string with length $|\epsilon| = 0$, and, for integers $k \geq 1$, $\Sigma^k$ is defined with

   $$\Sigma^k = \{xy : x \in \Sigma^{k-1} \text{ and } y \in \Sigma\}$$

Prove (by induction) that for every string $w \in \Sigma^k$, the length $|w| = k$.

4. Consider the finite automaton in Figure 1. Try the following types of strings on the FA to see if it accepts them.

   abbaabbbabaa, abaabbaabaa, babbbaba, bbaabbaabb

   What language do you think the FA may be accepting?

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Figure 1: state diagram of a finite automaton

The formal description of a DFA $M$ is

$$M = (\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$$

where $\delta$ function is given by the following table. Draw the state diagram of this machine.

<table>
<thead>
<tr>
<th>$q_i$</th>
<th>$u$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>$q_1$</td>
<td>$q_2$</td>
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<td>$q_2$</td>
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<td>$q_5$</td>
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</tbody>
</table>

6. Question 1.6 (a), (d), (j) on page 84 of the textbook (both 3rd edition and 2nd edition).

Given state diagram of DFAs recognizing the following languages. In all parts, the alphabet is $\{0, 1\}$.

a. $\{w : w$ begins with a 1 and ends with a 0$\}$

d. $\{w : w$ has length at least 3 and its third symbol is a 0$\}$

j. $\{w : w$ contains at least two 0s and at most one 1$\}$

NOTE: All homework answers need to be word-processed or typed. Hand-writing only applies to figure or table drawings. A hard copy of answers should be received in classroom or in the instructor’s office by 5:00pm on the due date (slide it under the door if the instructor is not in the office when you come). Policy on late homework answers is given in the syllabus.