This homework assignment is about NFA, NFA to DFA conversion, operations on regular languages, and regular expressions.

1. Design an NFA to recognize the following language, where $\Sigma = \{a, b, c\}$

   $L_1 = \{w : \text{the second last symbol of } w \text{ is not 'a'} \}$

   Answer:

   ![Diagram of NFA for $L_1$](credit: Connor Dooley)
2. Design an NFA to recognize the following language, where $\Sigma = \{a, b, c\}$

$L_2 = \{w : w \text{ contains an even number of } a\text{'s or contains the pattern '}aa'\}$

**Answer:**

![NFA Diagram](image)

*Figure 2: NFA for $L_2$ (credited: Connor Dooley)*
3. Based on the work for Questions 1 and 2, design an NFA to recognize each of the following languages:
   (a) $L_1 \cup L_2$;
   (b) $L_1L_2$;
   (c) $L_1^*$;

**Answer:**

(a)

![Figure 3: NFA for $L_1 \cup L_2$ (credited: Connor Dooley)](image)

Figure 3: NFA for $L_1 \cup L_2$ (credited: Connor Dooley)

(b)

![Figure 4: NFA for $L_1L_2$ (credited: Connor Dooley)](image)

Figure 4: NFA for $L_1L_2$ (credited: Connor Dooley)
Figure 5: NFA for $L_1^*$ (credited: Connor Dooley)
4. Consider the following NFA. Convert it to an equivalent DFA using the studied
method. Note the simple conversion process is to construct the extension set $E$ after
related transitions are determined. For example, if $R$ is a subset of states in the NFA
that has transition function $\delta$, then for symbol $x$, the new transition function
$\Delta(R, x)$ is computed as the result of the following sequence of steps:

Figure 6: state diagram of an NFA

(1)compute $\delta(r, x)$ for every $r \in R$;
(2)compute extension set $E(\delta(r, x))$ for every $r \in R$, if relevant $\epsilon$-transitions exist;
(3)take the union $\bigcup_{r \in R} E(\delta(r, x))$;

Draw the final DFA converted from the NFA.

Answer:
You would like to show all steps in computing the new transition function $\Delta(R, x)$ for
every $R$ and every $x$. Showing these steps serve two purposes:
(a) to reward you with partial credits even if your final answer may be wrong, and
(b) to remind you how you have come to the final answer.

(1) There are 8 possible states in the new DFA:

$\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}$

(2) The new transition $\Delta$ is defined as:

$\Delta(\{q_0\}, a) = \{q_0, q_1\}, \Delta(\{q_0\}, b) = \emptyset$
$\Delta(\{q_0, q_1\}, a) = \{q_0, q_1\}, \Delta(\{q_0, q_1\}, b) = \{q_0, q_1, q_2\}$
$\Delta(\{q_0, q_1, q_2\}, a) = \{q_0, q_1\}, \Delta(\{q_0, q_1, q_2\}, b) = \{q_0, q_1, q_2\}$

(3) The start state is $\{q_0, q_1\}$;
(4) The accept states are $\{q_0, q_1\}$ and $\{q_0, q_1, q_2\}$. 
5. For each of the following regular expressions, give two positive and two negative members for the language it generates:

(a) $a(ba)^*b$;
(b) $(\epsilon \cup b)a$;

**Answers:**

(a)
positive members: $ab$, $ababab$
negative members: $aba$, $\epsilon$

(b)
positive members: $a$, $ba$
negative members: $ab$, $bb$
6. Design an NFA for each of language given in Question 5.

Answers

(a)

![Figure 8: NFA for \(a(ba)^*b\) (credited: Connor Dooley)](image)

(b)

![Figure 9: NFA for \((\epsilon \cup b)a\) (credited: Connor Dooley)](image)

7. Give regular expressions for the following languages, where \(\Sigma = \{0, 1\}\)

(a) \(\{w : w \text{ contains exactly two } 0\text{'s }\}\)

(b) \(\{w : w \text{ contains at least two } 0\text{'s and at most one } 1\}\)

Hints: There are only a few ways that “exactly two 0’s” can be arranged in a string. “At least two 0’s” is the same as ”exactly two 0’s or more than two 0’s”.

answers:

(a) \(1^*01^*01^*\)

(b) \(000^* \cup 1000^* \cup 0100^* \cup 000^*10^*\)
8. In certain programming languages, comments appear between delimiters such as /# and #/. Let C be the language of all valid delimited comment strings. Such a string in C must begin with /# and end with #/ but have no intervening #/. For simplicity, assume the alphabet $\Sigma = \{a, b, /, \#\}$.

(a) Give an NFA that recognizes language C.

(b) Give a regular expression that generates C.

**Answers:**

(a)

(b) $/\#(a \cup b \cup / \cup (#*(a \cup b)))^*#/ $

**NOTE:** All homework answers need to be word-processed or typed. Hand-writing only applies to figure or table drawings. A hard copy of answers should be received in classroom or in the instructor’s office by 5:00pm on the due date. Policy on late homework answers is given in the syllabus. *Email submission will not be accepted unless a such a request has been approved.*