This homework assignment is about Turing machines, decidable languages, Turing recognizable languages, and (un)decidability

1. (20 points) Consider the following Turing machine, which recognizes language \( L = \{ w\#w : w \in \{0,1\}^* \} \).

   For each part in (a) and (b), give the sequence of configurations that the TM enters when started on the indicated string.
   
   (a) 10\#11  (b) 10\#10

2. (20 points) Design a Turing machine that recognizes the following language
   
   \( \{ w : w \in \{0,1\}^* \text{ contains at least two same symbols, with one at the end } \} \)

   You need to answer this question by taking the following steps:
   (1) give an outline for a TM that recognizes such strings \( w \);
   (2) draw a Turing machine diagram based on (1).
3. (20 points) A 2-PDA is a Push Down Automaton equipped with two stacks, in which the tops of the two stacks (instead of one stack) may be updated at each step. Prove that 2-PDAs are as powerful as Turing machines. (*Hint*: place the two stacks head-to-head to simulate the work tape of a Turing machine so the joint point of the two stacks simulates the position of the read-head of the TM.)

4. (20 points) Show that the class of decidable languages is closed under the complementation operation. Is the class of Turing recognizable languages also closed under all the above operations? Why?

5. (This question is for exercise only. You do not have to turn in your answers). Answer all parts for the following DFA $M$ and give reasons for your answers.

![DFA Diagram]

   (a) Is $\langle M, 0100 \rangle \in A_{DFA}$?  (b) Is $\langle M, 011 \rangle \in A_{DFA}$?  (c) Is $\langle M \rangle \in A_{DFA}$?
   (d) Is $\langle M, 0100 \rangle \in A_{REX}$?  (e) Is $\langle M \rangle \in E_{DFA}$?  (f) Is $\langle M, M \rangle \in EQ_{DFA}$?

6. (20 points) Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable. (*Hint*: use theorem 4.5 and theorem 1.54 (or lemma 1.55)).

7. (20 points) Let language $ALL_{DFA} = \{ \langle A \rangle : A \text{ is a DFA and } L(A) = \Sigma^* \}$. Show that $ALL_{DFA}$ is decidable. (*Hint*: use the known fact that the class of regular languages is closed under complement).

**NOTE:** All homework answers need to be word-processed or typed. Hand-writing only applies to figure or table drawings. **A hard copy of answers** should be received in classroom or in the instructor’s office by 5:00pm on the due date. Policy on late homework answers is given in the syllabus. **Email submission will not be accepted unless a such a request has been approved.**