There are seven questions with 140 points in total.

1. (20 points) (1) Run the Bellman-Ford algorithm on the directed graph of Figure 1, using vertex 4 as the source. Show \( d \) and \( \pi \) values after each of the \(|V| - 1\) passes. Assume edges are picked by the algorithm according to their lexicographical ordering, e.g., edge \((1, 3)\) is picked before edge \((2, 1)\), and edge \((2, 1)\) is picked before edge \((2, 3)\), etc..

2. (15 points) Give a simple example of a directed graph with negative weight edges for which Dijkstra’s algorithm produces an incorrect answer. Also argue that, if all negative weights occur only on the edges leaving the source \(s\), Dijkstra’s algorithm remains correct.

3. (15 points) Run the Floyd-Warshall algorithm on the graph given in Figure 1 to compute matrices \(D^{(2)}\) and \(D^{(3)}\).
4. (15 points) Consider the following decision version of MST problem:

**MST Decision:**

**Input:** an integer weighted graph $G = (V, E)$ and integer $k$;

**Output:** “yes” if and only if there exists a spanning tree with total weight $\leq k$;

Using the definition of class $\mathcal{NP}$ to prove that problem MST Decision is in the class $\mathcal{NP}$.

5. (20 points) Many optimization problems are closely related to their decision problem versions. Problem Maximum Independent Set is defined as follows:

**Input:** graph $G = (V, E)$;

**Output:** an independent set $I \subseteq V$ such that $|I|$ is the maximum;

A decision version of the problem is Independent Set:

**Input:** graph $G = (V, E)$ and integer $k > 0$;

**Output:** YES if and only if $G$ has an independent set $I$, $|I| \geq k$;

Prove that if Independent Set can be solved in polynomial time, so can Maximum Independent Set.

6. (15 points) Use the reduction from problem Independent Set to Clique to show the result of reduction for input instance $\langle G, 3 \rangle$, where $G$ is shown in the following Figure 2:

![Figure 2: Graph G for Q6.](image)

7. (40 points) Let $\Pi_1$ and $\Pi_2$ be two decision problems and $\Pi_1 \leq_p \Pi_2$. Explain why the following statements are true or not true:
(1) Either Π₁ ∈ \(\mathcal{NP}\) or Π₂ ∈ \(\mathcal{NP}\);
(2) If Π₂ ∈ \(\mathcal{NP}\), then Π₁ ∈ \(\mathcal{NP}\);
(3) If Π₁ is \(\mathcal{NP}\)-complete, then Π₂ is \(\mathcal{NP}\)-complete;
(4) Π₂ ≤ₚ Π₁;

The answers must be word-processed or typed. You may substitute formulae and figures with hand-writings. Your submitted algorithms should be in the pseudo-code, not in any specific programming language. Answers deviating from these requirements will be returned without grading.

The answers must be the student’s own work. Idea sharing and referencing to others’ work are not allowed. Plagiarism and other forms of academic dishonesty will be handled within the guidelines of the Student Handbook and reported to the University.