Homework No. 6
CSCI 4470/6470 Algorithms, CS@UGA, Spring 2018

Due Tuesday April 30, 2018 (by 11:00am in classroom)

The answers must be word-processed or typed. You may substitute formulae and figures with hand-writings. Your submitted algorithms should be in the pseudo-code, not in any specific programming language. Answers deviating from these requirements will be returned without grading.

The answers must be the student’s own work. Idea sharing and referencing to others’ work are not allowed. Plagiarism and other forms of academic dishonesty will be handled within the guidelines of the Student Handbook and reported to the University.

There are 100 points in total.

1. (20 points) Consider the 2-SAT problem:

2-SAT

Input: a boolean formula \( f(x_1, \ldots, x_n) \) in conjunctive form with exact two literals in every clause;

Output: “yes” if and only if \( f(x_1, \ldots, x_n) \) is satisfiable.

2-SAT is a restricted version of the SAT problem. It has formula instances with exactly two literals in every clause (here a literal is either a boolean variable or its negative form). For example,

\[
f(x_1, \ldots, x_n) = (\neg x_1 \lor x_3) \land (x_2 \lor \neg x_4) \land (x_1 \lor x_2) \land \ldots
\]

Unlike SAT, which is NP-complete, 2-SAT has polynomial-time algorithms. But this question asks you to prove that 2-SAT is in the class NP, using the definition of NP.

2. (20 points) Consider the following decision version of MST problem:

MST Decision:
INPUT: an integer weighted graph $G = (V, E)$ and integer $k$;
OUTPUT: “yes” if and only if there exists a spanning tree
with total weight $\leq k$;

Using the definition of class $\mathcal{NP}$ to prove that problem MST Decision is in the class $\mathcal{NP}$.

3. (20 points) Use the reduction from problem Independent Set to Clique to show the result of reduction for input instance $(G, 3)$, where $G$ is shown in the following Figure:

4. (20 points) Use the reduction from problem Vertex Cover to Independent Set to show the result of reduction for input instance $(G, 3)$, where graph $G$ is shown in Q3 above.

5. (20 points) Let $\Pi_1$ and $\Pi_2$ be two decision problems and $\Pi_1 \leq_p \Pi_2$. Explain why the following statements are true or not true:

   (1) either $\Pi_1 \in \mathcal{NP}$ or $\Pi_2 \in \mathcal{NP}$;
   (2) If $\Pi_2$ is polynomial time solvable, so is $\Pi_1$;
   (3) If $\Pi_2$ is $\mathcal{NP}$-complete, then $\Pi_1$ is $\mathcal{NP}$-complete.