Problem 1
Design a 2 bit comparator circuit with 2 bit numbers \( A = A_1A_0 \) and \( B = B_1B_0 \) as inputs and 3 outputs indicating whether \( A = B \), \( A < B \) or \( A > B \). Show the truth table, derive the Boolean function in SOP form and implement the logic using a Programmable Logic Array (PLA).

Problem 2
Build a full adder using two half adders and other necessary gates.

Problem 3
Assume you are given a 3-bit number \( X = X_2X_1X_0 \). Derive the Boolean functions in SOP form that are true if only if:
- a. \( X \) contains only one 1
- b. \( X \) contains an even number of 1’s
- c. \( X \) is a number less than 3

Problem 4
Implement each of the Boolean functions in Problem 3 using a PLA.

Problem 5
Implement a switching network that has two data inputs \( A \) and \( B \), two data outputs \( C \) and \( D \) and a control input \( S \). When \( S = 0 \) the network is in pass-through mode, that is, \( C = A \) and \( D = B \). When \( S = 1 \) the network is in crossing mode, that is, \( C = B \) and \( D = A \). (Hint: Use multiplexors in your design).

Problem 6
Consider a priority encoder with 4 inputs \( X_3, X_2, X_1, \) and \( X_0 \) and 2 outputs \( Y_1 \) and \( Y_0 \). If one of the inputs goes high, for example \( X_2 \) then the binary code corresponding to the position of the input appears at the output. In this example the output would be \( Y_1Y_0 = (10)_2 = (2)_{10} \). However, if more than one input goes high at the same time then the binary code corresponding to the highest priority position appears at the output. For example if \( X_3 \) and \( X_2 \) are simultaneously high and if input \( X_3 \) is assumed to have a higher priority than \( X_2 \) then the output will be \( Y_1Y_0 = (11)_2 = (3)_{10} \). Derive the truth table and the Boolean expressions in SOP form for the 4:2 priority encoder and assume that input \( X_3 \) has the highest priority and \( X_0 \) the lowest, i.e., \( \text{priority}(X_3) > \text{priority}(X_2) > \text{priority}(X_0) \). (Hint: Use logic don’t cares to simplify the truth table).