## CSCI 6610, Spring 2014 (Rod Canfield)

**Text**: *Intro. to the Theory of Computation* by M. Sipser. **Syllabus**: Terminology, background, and proofs of the following ...

**Theorem 1.** (Myhill, Nerode) A language L is regular if and only if the associated equivalence relation  $R_L$  has finitely many equivalence classes.

**Theorem 2.** (Hennie) Let L be the language over the alphabet  $\{a, b, c\}$  defined by

 $L \stackrel{\text{def}}{=} \{wc^{|w|}w^{\text{rev}} : w \in \{a, b\}^*\}.$ 

If T is a deterministic Turing machine with q states that recognizes L, then for each integer  $n \ge \log_2 q$  there is a string  $w \in L$  of length 3n such that on input w the machine T executes at least  $n^2/\log q$  steps before halting.

**Theorem 3.** (Shannon) For each  $\epsilon > 0$ , almost all Boolean functions of n variables have circuit complexity greater than  $(1 - \epsilon)2^n/n$ .

**Theorem 4.** (Church) The theory  $(\mathcal{N}, +, \times)$  is undecidable.

**Theorem 5.** (Jerrum, Sinclair, Vigoda) There is a fully polynomial randomized approximation scheme for computing the permanent of an  $n \times n$  matrix with non-negative entries.

**Theorem 6.** (Savitch) For  $f(n) \ge \log n$ ,

$$NSPACE(f(n)) \subseteq SPACE(f(n)^2)$$

**Theorem 7.** (Immerman, Szelepcsényi) NL = coNL.

Theorem 8. (Shamir) IP=PSPACE.

**Theorem 9.** (Baker, Gill, Solovay) For suitable oracles A and B we have

$$P^{A} = NP^{A}$$
$$P^{B} \neq NP^{B}$$

**Theorem 10.** (Agrawal, Kayal, Saxena)  $PRIMES \in P$ .