New HW & Reading

HW #4, due Tues 9/13, Chapter 5, #3, 9, 11, 18, 25, 47.

Graduate students:
(1) prove the Theorem given below RE LR minima
HINT: Start with a permutation having \( k \) cycles. Locate the smallest element in each cycle, and write the cycle in one-line notation. Continue.
(2) Determine the \( \# \) ways to tile a \( 1 \times n \) board using dominoes and monomers. (A monomer is a single square) Same question for the \( 2 \times n \) board.

Reading for September 13, pages 161–171; for September 15, pages 172–182.

From Last

Suppose a matrix \( A \) whose entries are nonnegative integers satisfies:

\[
A^6 = 4A^3 - I
\]
\[
A^7 = 4A^4 - A
\]
\[
\vdots
\]
\[
A^m = 4A^{m-3} - A^{m-6}
\]
\[
\vdots
\]

Let \( a_m \) be the \((0,0)\) entry of \( A^{3m} \); then, \( a_m = 4a_{m-1} - a_{m-2} \) for \( m \geq 2 \). We have an application where \( A \) is the incidence matrix of a directed graph which corresponds to placing dominoes on a \( 3 \times m \) board.

Combinatorial proof: \( k^n \binom{n}{k} = \binom{n-1}{k} \); implies

\[
\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}.
\]

Also gave calculus based proof, using derivative.

Today

The latter method can handle the sum of \( k^p \binom{n}{k} \) for \( p \) larger than 1. Other topics from the chapter on binomial coefficients; e.g. Newton’s theorem.

Handout 4 addenda
(1) We also learned on 8/30 how to generate RANDOM permutations. We never mentioned the item “LR minimima.” A permutation \( \sigma_1 \ldots \sigma_n \) written in one-line notation has a certain number of left-to-right minima; these are \( \sigma_i \) which are smaller than all \( \sigma_j \) with \( j < i \). (The first, \( \sigma_1 \), is counted by default.)

Theorem: Let \( n \geq 1 \). The permutations of \([n]\) which have \( k \) LR minima are in 1-1 corrsp with those having \( k \) cycles.
Solutions to some homework #2 problems (Chapter 2)

#6 94,830
#7 241,920 = 8! \cdot 3!
#9a 14! - 2(13!)
#9b 14! - 13!
#13b \binom{50}{25} \binom{50}{35}
#14 \frac{5!}{3!2!}
#45a \binom{24}{3}
#45b 5^{20}
#45c 24! / 4!
#46a \binom{20}{n} n! / 4!
#46b \binom{2n+1}{2}
#47 \binom{2n+3}{2} - 3 \binom{n+2}{2} = (n + 1)n / 2
#55a \binom{51}{36} / \binom{34}{2}
#59a 10 \cdot \binom{4}{3} \cdot 9 \cdot \binom{4}{2} \text{ divided by } \binom{40}{5}
#59b 7 \cdot (4! - 4) \text{ divided by } \binom{40}{5}
#59c 7 \cdot 4 \text{ divided by } \binom{40}{5}
#59d \binom{10}{4} \binom{5}{2} \cdot 32 \text{ divided by } \binom{40}{5}
#59e \binom{10}{3} - \binom{30}{5} \text{ divided by } \binom{40}{5}
#60 \text{ “at random” confusing}
#61 \text{ first part: } 1 / \binom{9}{5}
#61 \text{ second part: } 1 / \binom{9}{5}^2

undergrads: 20 answers at 3 each, 60 total
grads: 20 at 3, plus 2 proofs at 5 each, 70 total