New HW & Reading
HW #5, due Thurs 9/22, Chapter 6, # 5, 17(BUT with 3 c’s instead of 2), 25, 29(the people are distinguishable), 33.

Graduate students:
(1) By taking a derivative and comparing coefficients of $z^n$, find a recursion for the sequence of integers $a_n$ defined by
\[
\exp \left( z + \frac{z^2}{2} \right) = \sum_{n=0}^{\infty} a_n \frac{z^n}{n!}.
\]
Show that $a_n$ equals the number of involutions of $[n]$. (An involution is a permutation which equals its own inverse.)

(2) In the same manner, find a recursion for $d_n$, where
\[
\exp \left( \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4} + \cdots \right) = \sum_{n=0}^{\infty} d_n \frac{z^n}{n!}.
\]
Show that $d_n$ equals the number of derangements of $[n]$.
Hint. Rewrite the left side as
\[
\frac{e^z}{1 - z}.
\]

Reading for September 13, pages 161–171; for September 15, pages 172–182.

From Last We gave a combinatorial proof for
\[
\sum_{n=k}^{N} \binom{n}{k} = \binom{N + 1}{k + 1}.
\]
We went over Newton’s binomial theorem; Brualdi gives the date 1676. We proved
\[
(1 + z)^{1/2} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2n}{2n - 1} \binom{2n}{n} \left( \frac{z}{4} \right)^n.
\]
Several times we have used “generating functions” to obtain algebraic (as opposed to combinatorial proofs) of identities. E.g., $\sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right)^2$. How about
\[
\sum_{\ell} \binom{m}{k - \ell} \binom{n}{\ell} = \binom{m + n}{k}.
\]
Combinatorial proof.

**Today**  Any questions from HW solutions on last handout? Note our grader’s name and email address: Sal Lamarca, slamarca@uga.edu

Start Chapter 6, PIE. Good way to think: we have a collection $O$ of combinatorial objects, and a collection of properties $P_1, P_2, \ldots$ which the objects are capable of having. Example. Consider index cards that contain pairs $(X, S)$ where $X$ is an object and $S$ is a set of properties that $X$ has. (It may not be all the properties.) Let $N_i$ be the number of cards in which the set $S$ has size $i$. Then, the number of objects with no properties is $N_0 = N_1 + N_2 - \ldots$.

**Handout 5** We neglected to tell how to sum $k^p \binom{n}{k}$ for $p > 1$.  