Please take time to evaluate the course (comments greatly appreciated) by logging into
the course evaluation system using your MyID and password. Link:

eval.franklin.uga.edu

CSCI/MATH 4670/6670. The course evaluation system is on-line beginning November 29
(Tuesday) and ending December 7 (Wednesday) at 11.59pm.

**From Thurs 11/17** We started w/ the defn of a STS (Steiner Triple System), and proved
that there is a unique (up to isomorphism) one with \(v = 7\). We showed that this STS(7)
is also a projective plane, known as the Fano plane. It is the only design which is both an
STS and a projective plane. (It is also the first “backtrack-free STS”; others known, so
far, for \(v = 15, 31, 63, 127, 255, 511\).)

We proved that \(v \equiv 1, 3 \mod 6\) is an obvious necessary condition for the existence of an
STS(\(v\)). (This comes about because \(\binom{v}{2}\) must be divisible by 3, and \(v\) must be odd.) We
stated, but did not prove

**Theorem.** The obvious necessary conditions for the existence of an STS on \(v\) points are
also sufficient.

The number of distinct (up to isomorphism) STS’s on \(v = 1, 3, 7, 9, 13, 15\) points are
1, 1, 1, 1, 2, 80. The number on \(v = 19\) points is quite large, and was found only relatively
recently. Beyond 19, exact counts are not known.

We showed that STS’s are a special case of **balanced incomplete block designs**, aka BIBD’s.
The latter have four parameters \(b, k, v, \lambda\) which are: number of blocks, size of blocks, size
of ground set, and number of times each pair is covered. The incidence matrix \(A\) is \(b \times v\),
with entry \((i, j)\) equal to 1 iff block \#\(i\) contains element \#\(j\). The row sums of this matrix
are all \(k\), by definition. We proved that the column sums are also all equal, and have the
common value \(\lambda(v - 1)/(k - 1)\). (Hence, integrality of the latter is a necessary condition
for the existence of a BIBD(\(b, k, v, \lambda\)).)

Finally, we called attention to an interesting theorem of the text, which we do not have
time to study in detail, that proves we must have \(b \geq v\) in a BIBD. What is interesting
about the proof is that it uses linear algebra, not combinatorics.

During the HW return question period, a question came up about the Catalan number
problem which I was unable to answer at the time, so we should start today with that.
(As well as questions about HW due Thurs)

And then we should move on to group actions on combinatorial objects.

- Use STS(\(v\)) as example of automorphism, or symmetry
- Look also at binary necklaces
- Burnside’s Lemma – state
- Count necklaces using Burnside’s Lemma
Key post-midterm topics: (go back to Handout 11 for first reading)

1. ordinary and exponential generating functions; including: \( t_n, r_n \), balls in cells, integer partitions.
   2. Stirling numbers of first and second kinds
   3. Catalan numbers
   4. BIBD’s, esp Steiner triple systems
   5. Counting up to symmetry, necklaces, Burnside Lemma