Reading

April 23: Sections 8.1 and 8.2
April 25: Sections 8.3, 8.4, and 8.5 (Note: 8.5 was accidentally omitted from the listing on Handout 14.)

Homework #15. There is no more written homework! But here are some final exam notes:

1. the final takes place in our regular room, 12N-3p, on Tue, May 7.
2. usual format; exam is comprehensive.
3. reg office hours this week; also next week; also one special last minute office hours on Mon, May 6, from 10 til Noon.

Fourteenth week summary

Tue 4/16. Drew a picture of Petersen’s graph embedded in a torus. We reviewed both uses of the word genus, mentioning \( \chi(S) \) for a surface \( S \), and \( \chi(G) \) for a graph \( G \).

Thu 4/18. Went over two applications of graph coloring. Reviewed a little from Tuesday, reminded all of time/date for Final, and mentioned three remaining topics to be covered in class:

1. \( k \)-partite graphs and Turán graphs, \( T_k(n) \)
2. Brooks’ Theorem
3. Vizing’s Theorem

We defined \( T_k(n) \), and worked out the number of edges in it, which is denoted \( t_k(n) \). We then proved Observation 8.16, which says that \( t_k(n) \) is the maximum number of edges in a simple graph \( G \) satisfying \( |V(G)| \leq n \) and \( \chi(G) \leq k \). Moreover, \( T_k(n) \) is the only graph achieving this maximum. Then we went on to Theorem 8.18. This theorem includes Observation 8.18, and adds to it as follows: \( t_k(n) \) is also the maximum number of edges in a simple graph \( G \) satisfying \( |V(G)| \leq n \) and not
having $K_{k+1}$ as a subgraph. Moreover, $T_k(n)$ is the only graph achieving this maximum.

We then went hurriedly through the subject matter of each of the four undergrad HW problems due this coming Thursday. And, finally, we introduced Brooks’s Theorem. We also gave a small amount of biographical information about R. L. Brooks. The Theorem has to do with the fairly trivial bound (proven twice now: Thursday 4/12 and Tuesday 4/16)

$$\chi(G) \leq \Delta(G) + 1.$$ 

We observed that for $G$ equal to either a complete graph or an odd cycle, one has equality in $(\ast)$. But, among all simple connected graphs, those two are the only cases of equality; that is, if $G$ is simple and connected, not a complete graph, and not an odd cycle, then

$$\chi(G) \leq \Delta(G).$$

The proof is the first order of business today, Tuesday 4/23.