Last time. Planar graphs, Jordan curve theorem, sphere vs plane, Euler’s formula, $K_5$ not planar.

Today. Kuratowski’s theorem.

Homework # 11. Due Thursday, 4/21. Text: 7.1, 7.2, 7.3; plus the following two.

4. Let $G$ be a bipartite graph with parts $A$ and $B$. Assume that $G$ is bi-regular; that is, each vertex in $A$ has the same degree, and each vertex in $B$ has the same degree. Prove that if $|A| \leq |B|$, then $A$ can be matched into $B$. (see Handout 15) Guide to proof:
   (a) Let $d_A$ be the degree of each vertex in $A$, and $d_B$ be the degree of each vertex in $B$. Show that $d_A \times |A| = d_B \times |B|$.
   Conclude that $d_A \geq d_B$.
   (b) Let $S \subseteq A$; show that the number of edges leaving $S$ is $d_A \times |S|$. Show that for each $b \in \Gamma(S)$ the number of edges entering $b$ from $S$ is at most $d_B$. Conclude that $d_A \times |S| \leq d_B \times |\Gamma(S)|$
   (c) From part (b) conclude $|S| \leq |\Gamma(S)|$
   Done.

5. Suppose $p$ people are placed, with some overlap, into $c$ committees. Each committee has the same size, and each person serves on the same number of committees. Prove that so long as $p \geq c$ then each committee can elect/appoint a chairperson in such a manner that nobody is required to chair two different committees.

Graduate / Bonus

6. (Graduate/bonus). Let $S = \{1, 2, \ldots, n\}$. Form a bipartite graph with parts $A$ and $B$. Let $A$ consist of all subsets of $S$ of size $k$, and $B$ consist of all subsets of $S$ of size $k + 1$. Two subsets $X, Y$ are joined by an edge if and only if $X \subseteq Y$. Show that if $k < n/2$, then $A$ can be matched into $B$.