Review. from Tues 1/18 last class: Defn of digraph, in- and out-degrees; defn of walk, trail, path; defn of connected graph, and connected components of a graph.

Review. from Thurs 1/20, the last class: We finished the proof of Theorem 2.15 telling the maximum number of edges in a graph having $n$ vertices and $k$ components. We stated and proved “Adam’s theorem,” aka the “Handshaking Theorem,” from page 18. We discussed several ways of building new graphs from old: vertex deletion, edge deletion, edge contraction. For the latter (edge contraction) there is one version for general graphs and another for simple graphs. The second, for simple graphs, keeps you within the realm of simple graphs. If we start with a simple graph $G$, and perform a series of simple edge contractions, we end up at a smaller simple graph $G'$. There is another way to describe $G'$ that involves partitioning the vertices. This process of obtaining $G'$ leads to the notion of minor: $G'$ is a minor of $G$, denoted $G' \preceq_m G$. This relation gives a partial order on the set of simple graphs. A deep theorem of Robertson and Seymour says that within every infinite set of simple graphs there must be a pair which are $\preceq_m$ related.

Topics for today. Weakly connected digraphs, strongly connected digraphs, automorphism group, labelings, trees.

New Readings.
For January 27: Sections 3.2, 3.3, and 3.5
For February 1: Sections 4.1 and 4.2
For February 3: Sections 4.3 and 4.4

Homework #3. (1) Draw all trees up to 5 vertices. For each count the labelings. Confirm that the number of labelings adds to $n^{n-2}$ as predicted.
Hand in exercises 2.15, 2.22*, 2.24, 2.26*, 2.30*, 3.3.
Graduate/bonus exercises 2.23*, 3.4.

Notes
2.22 should read “... every regular simple graph is regular.”
2.23 should read “Show that if a simple graph \( G \) on \( n > 1 \) vertices ...
2.26 The second sentence should read “Viewing these paths as subraphs of \( G \), show that \( p_1 \Delta p_2 \) constitutes an edge-disjoint union of one or more cycles, possibly along with some isolated vertices.”
2.30 should read “simple contraction” instead of “contraction” in both places.