Homework 10. Due Tue 10/30.

1. Exercise 7.4.
2. Exercise 7.5.
3. Exercise 7.10.
4. Exercise 7.11.

Tenth week recap

Tue 10/16. Summarized 2-SAT from last time: if you don’t find a solution in $2mn^2$ steps, then stop and declare the instance unsatisfiable. The probability you are wrong is bounded above by $2^{-m}$. (Markov inequality used to obtain latter.)

We moved on to 3-SAT, with $k$-SAT promised for homework. We are led to consider a random walk on the integers $0, 1, \ldots, n$ with probability of going left and right $2/3, 1/3$. This time we find $h_j = 2^{n+2} - 2^{j+2} - 3(n - j)$. There results a $\Theta(2^n)$ algorithm – no good since we could have exhausted and gotten that. New idea: limit the time spent walking, and then restart. We went through the analysis that shows this approach (with a $3n$ limit on steps before restart) leads to a $n^{3/2}(4/3)^n$ algorithm.

Wed 10/17. Will reported on Brandon’s and his connectivity program. Key feature of note: use of linear congruential generator. This means random value $x_{i+1}$ is given by the formula

$$x_{i+1} = ax_i + c \mod N.$$  

The three values $a, c, N$ are parameters. They take $N = n \times (n - 1)$, twice the number of edges. LCG theory says how to choose $a, c$ so that (*) is full period. Half of the values $1, 2, \ldots, N$ can naturally be
interpreted as edges \((i, j)\). LCG theory says there will be no repeats. The assumption: by varying the starting point \(x_0\) we get a “nice” sample of the \(\binom{n}{2}\) possible permutations.

Thu 10/18. Reported on program results that does the following: choose a starting position \(j, 0 \leq j \leq n\), according to the binomial. Now move left and right with probabilities \(2/3, 1/3\), stopping if you reach \(n\). Restart after \(3n\) moves. The value \((\text{Average})^{1/n}\) seems to be decreasing to a limit. What is all this related to?

The rest of Thursday was DEFINITION DAY. Here are some of the things we covered: notations \(P_{i,j}, P^t_{i,j}\) accessibility, communicating, irreducible, recurrent, transient, positive recurrent, null recurrent. Return probabilities \(r^t_{i,j}\) and expected hitting time (return time) \(h_{i,j}\). Gave an example of a MC on the positive integers with state 1 being null recurrent. Stated Lemma 7.5, and also the Gambler’s Ruin theorem.

Eleventh week topics


Wed 10/24. Finish stationary, state Theorem 7.7,

Thu 10/25. Random walk on an undirected graph.