Homework 3. Due Tue 9/11.

1. Exercise 3.6

2. Exercise 3.8. Hint: Let $X$ denote the r.v. which measures the run time. Since $\mathbf{E} X = O(n^2)$, there is a constant $C$ s.t. $\mathbf{E} X \leq Cn^2$.
   (a) What is Markov’s bound on $\Pr(X \geq Cn^2)$?
   (b) Why does that imply that there is no $\omega$ s.t. $X(\omega) \geq Cn^22^n$?
   (c) So, ...

3. Exercise 3.10.

4. When have we used Exercise 3.14 in class?

5. Exercise 3.22.

Third week recap Tuesday: (1) We finished up Exercise 3.21, on random permutations. Pointed out validity of Poisson approximation for the number of fixed points in a random permutation. (2) Went over the general idea of a Chernoff bound: start with

\[
\Pr(X \geq x) = \Pr(e^{tX} \geq e^{tx}), \quad t > 0,
\]

bound the right hand side using Markov’s inequality, then set the free parameter $t$ to make the result as strong as possible. (3) We carried this out for $X = \sum X_i$, with the $X_i$ iid, each being $\pm 1$ with probability $1/2$. (4) Explained how latter could be applied in other cases by a change of variable. (5) [side topic] talked a little about Persi Diaconis’ theory of re-enforced random walks.

Wednesday: Gave the background and definitions for the hypercube, and what it means to send a set of messages whose destinations are given by a permutation. Talked about the idea of bit-fixing as a way of routing, and also the need for buffer management. Made sure we understand what is being asked by Exercise 4.21.
Thursday: We continued w/ the packet-switching problem, solving Exercise 4.21, and then moving on to the Theorem. The theorem states that with probability exceeding $1 - O(N^{-1})$, the randomized algorithm in which each packet is given a random intermediate destination will achieve run-time $O(\log N)$. (Exercise 4.21 shows that the simple bit-fixing scheme can take time $\Omega(N^{1/2})$ for certain unlucky permutations.)

We discussed the main random variables that are used in the proof: $T_1(M), X_1(e), H_k, H$. We defined what a possible path $P$ is. We worked with three key equations,

$$T_1(M) \leq \sum_{i=1}^{m} X_1(e_i),$$

$$T_1(P) = \sum_{i=1}^{m} X_1(e_i),$$

$$\Pr(T_1(M) > T) \leq \sum_P \Pr(T_1(P) > T).$$

There are $2^2 \times 2^n = 2^{2n}$ possible paths $P$. The strategy of the proof is to bound $\Pr(T_1(P) > 30n)$ by $2^{-3n}$. This is done by treating two cases: $H < 6n, H \geq 6n$. Two things remain to be shown:

$$\Pr(H \geq 6n) \leq 2^{-6n},$$

$$\Pr(T_1(P) > 30n \mid H < 6n) \leq 2^{-3n-1}.$$

When we stopped, we were still in the process of understanding the random variable $H = \sum_{k=0}^{N-1} H_k$ which is used for conditioning.

**Fourth week topics**

- Tues 9/4. Finish proving the Theorem. Intro to Chapter 5.
- Thurs 9/6. Finish Bloom; start Ham cycle algorithm.