Homework #2. Due Tue 9/4.
#1 – Exercise 2.2.8, page 40.
#2 – Prove property (i) of matrix multiplication, page 41. (associativity)
#3 – Exercise 2.3.1, page 47.
#4 – Exercise 2.4.8, page 58.
#5 – Exercise 2.6.7, page 65. (Hint: show that the product $U_1U_2$ preserves inner product.)

Note Remember that equations 2.103 and 2.104 are modified in the text errata so that $\langle c\vec{v}, \vec{w} \rangle = c\langle \vec{v}, \vec{w} \rangle$. It might be advisable to pencil in the corrections from the 17-page errata. (see previous handout)

Second week summary We were focused on Chapter 2, a review of linear algebra. All our vector spaces and matrices are over the complex numbers. We proved this in its entirety:

**Theorem.** The following are equivalent for a finite matrix $U$ over the complex numbers:

(i) $\| U\vec{v} \| = \| \vec{v} \|$ for all vectors $\vec{v}$. ($U$ preserves length)

(ii) $\langle U\vec{v}, U\vec{w} \rangle = \langle \vec{v}, \vec{w} \rangle$ for all pairs of vectors $\vec{v}, \vec{w}$. ($U$ preserves inner product)

(iii) $U^\dagger U = I$. ($U$ is unitary; its inverse equals its conjugate-transpose.)

(iv) The columns of $U$ form an orthonormal system.

So, a unitary matrix is one which satisfies one, whence all four, of the above. We found the following examples of $N \times N$ unitary matrices: the identity matrix, any permutation matrix, any column-permutation of a unitary matrix, any row-permutation of a unitary matrix, the term-by-term conjugate of a unitary matrix, the transpose of a unitary matrix (see Note, below), the dagger of a unitary matrix, and any matrix of the form $e^{iY}$, where $Y$ is an $N \times N$ Hermetian matrix. A Hermetian matrix is one which satisfies $Y = Y^\dagger$. We proved this formula for the dagger
of a product: \((AB)^\dagger = B^\dagger A^\dagger\). One corollary of the product formula is:

\[(iA)^\dagger = -iA^\dagger.\]

**Note.** We did not prove that the transpose of a unitary matrix is unitary, but we observed that it follows from the following: if the \(N \times N\) matrix \(A\) has a left inverse \(B\), then \(B\) is also a right inverse for \(A\). In symbols,

\[BA = I \implies AB = I.\]

We also identified one additional \(N \times N\) unitary matrix which is not contained among the above examples. The matrix \(F_N\) defined by

\[F_N = \frac{1}{N^{1/2}}(a_{jk}),\]

where

\[a_{jk} = e^{2ijk\pi/N}.\]

This matrix defines the discrete Fourier transform.

By way of arriving at the matrix \(e^{iY}\) above, we reviewed the definition and properties of the exponential function in the complex domain. This allowed us to say what are the \(N\) \(N\)th roots of 1 in the complex plane. Finally, we stated the Fundamental Theorem of Algebra.

**Third week topics**

Tues 8/28. The discrete Fourier transform as an operator on polynomials. Define eigenvalues, eigenvectors, and the characteristic polynomial. For a Hermitian operator, why are the eigenvectors associated with distinct eigenvalues perpendicular? Proposition 2.6.4, page 64. End classroom coverage of Chapter 2 with Definition 2.4.9, page 60. Other things needed from Chapter 2, for example, tensor products, will be discussed as they arise. Time permitting, (not terribly likely), commence Chapter 3.