1. [20 points][MID] The \textit{subset}_{21} problem is stated as follows. Given a set of \( N \) positive integers \( X = \{x_1, x_2, \ldots, x_n\} \). Find a subset \( P \) of the set \( X \) such that the sum of the elements of \( P \) is equal to 21. For example, if \( N=5 \) and the set \( X = \{12, 17, 3, 24, 6\} \), the set \( P = \{12, 3, 6\} \) is a valid solution for the \textit{subset}_{21} problem in this example.

Formulate the \textit{subset}_{21} problem as a Genetic or Evolutionary Algorithm optimization. You may use binary representation, OR any representation that you think is more appropriate. you should specify:

- A representation.
- A fitness function. Give 3 examples of individuals and their fitness values if you are solving the above example (i.e. \( X = \{12, 17, 3, 24, 6\} \)).
- A set of mutation and/or crossover and/or repair operators. Intelligent operators that are suitable for this particular domain will earn more credit.
- A termination criterion for the evolutionary optimization which insures that you terminate with a valid solution for the \textit{subset}_{21} problem if possible without running indefinitely.

2. [20 points][MID] The \textbf{graph k-coloring} problem is stated as follows: Given an undirected graph \( G = (V, E) \) with \( N \) vertices and \( M \) edges and an integer \( k \). Assign to each vertex \( v \) in \( V \) a color \( c(v) \) such that \( 1 \leq c(v) \leq k \) and \( c(u) \neq c(v) \) for every edge \( (u, v) \) in \( E \). In other words you want to color each vertex with one of the \( k \) colors you have and no two adjacent vertices can have the same color.

For example, the following graph can be 3-colored using the following color assignments: \( a=1,b=2,c=1,d=2,e=3,f=2,g=3 \)

\begin{center}
\begin{tikzpicture}[every node/.style={circle,draw}]
    \node (a) at (0,0) {a};
    \node (b) at (1,0) {b};
    \node (c) at (2,0) {c};
    \node (d) at (1,-1) {d};
    \node (e) at (2,-1) {e};
    \node (f) at (3,-1) {f};
    \node (g) at (2,1) {g};
    \draw (a) -- (b) -- (c); % a --- b --- c
    \draw (a) -- (d); % a \ / \ b
    \draw (b) -- (d); % \ / \ b
    \draw (c) -- (e); % c --- e
    \draw (c) -- (f); % c --- f
    \draw (d) -- (g); % d --- g
\end{tikzpicture}
\end{center}

Formulate the \textbf{graph k-coloring} problem as an evolutionary optimization. You may use a vector of integer representation, OR any representation that you think is more appropriate. you should specify:

- A representation.
3. [20 points][FIN]

The **minimum vertex cover** problem is stated as follows: Given an undirected graph \( G = (V, E) \) with \( N \) vertices and \( M \) edges. Find a minimal size subset of vertices \( X \) from \( V \) such that every edge \( (u, v) \) in \( E \) is incident on at least one vertex in \( X \). In other words you want to find a minimal subset of vertices that together touch all the edges.

For example, the set of vertices \( X = \{a,c\} \) constitutes a minimum vertex cover for the following graph:

```
  a   ---b---c   ---g
  / \      |      |
  / \      |      |
  d   e   f
```

Formulate the **minimum vertex cover** problem as a Genetic Algorithm or another form of evolutionary optimization. You may use binary representation, OR any representation that you think is more appropriate. you should specify:

- A representation.
- A fitness function. Give 3 examples of individuals and their fitness values if you are solving the above example.
- A set of mutation and/or crossover and/or repair operators. Intelligent operators that are suitable for this particular domain will earn more credit.
- A termination criterion for the evolutionary optimization which insures that you terminate with a valid solution to the **minimum vertex cover** problem if possible without running indefinitely.