1. [20 points][MID] The subset$_{21}$ problem is stated as follows. Given a set of $N$ positive integers $X = \{x_1, x_2, \ldots, x_n\}$. Find a subset $P$ of the set $X$ such that the sum of the elements of $P$ is equal to 21. For example, if $N=5$ and the set $X = \{12, 17, 3, 24, 6\}$, the set $P = \{12, 3, 6\}$ is a valid solution for the subset$_{21}$ problem in this example.

Formulate the subset$_{21}$ problem as a Genetic or Evolutionary Algorithm optimization. You may use binary representation, OR any representation that you think is more appropriate. you should specify:

- A fitness function. Give 3 examples of individuals and their fitness values if you are solving the above example (i.e. $X = \{12, 17, 3, 24, 6\}$).
- A set of mutation and/or crossover and/or repair operators. Intelligent operators that are suitable for this particular domain will earn more credit.
- A termination criterion for the evolutionary optimization which insures that you terminate with a valid solution for the subset$_{21}$ problem if possible without running indefinitely.

2. [20 points][MID] The graph k-coloring problem is stated as follows: Given an undirected graph $G = (V, E)$ with $N$ vertices and $M$ edges and an integer $k$. Assign to each vertex $v$ in $V$ a color $c(v)$ such that $1 \leq c(v) \leq k$ and $c(u) \neq c(v)$ for every edge $(u, v)$ in $E$. In other words you want to color each vertex with one of the $k$ colors you have and no two adjacent vertices can have the same color.

For example, the following graph can be 3-colored using the following color assignments: $a=1,b=2,c=1,d=2,e=3,f=2,g=3$

```
   a---b---c---g
  / \    |
 /   \   |
 d --- e f
```

Formulate the graph k-coloring problem as an evolutionary optimization. You may use a vector of integer representation, OR any representation that you think is more appropriate. you should specify:

- A representation.
- A fitness function. Give 3 examples of individuals and their fitness values if you are solving the above example.
• A set of mutation and/or crossover and/or repair operators. Intelligent operators that are suitable for this particular domain will earn more credit.

• A termination criterion for the evolutionary optimization which insures that you terminate with a valid solution to the graph k-coloring problem if possible without running indefinitely.

3. [20 points][FIN]

The minimum vertex cover problem is stated as follows: Given an undirected graph \( G = (V, E) \) with \( N \) vertices and \( M \) edges. Find a minimal size subset of vertices \( X \) from \( V \) such that every edge \( (u, v) \) in \( E \) is incident on at least one vertex in \( X \). In other words you want to find a minimal subset of vertices that together touch all the edges.

For example, the set of vertices \( X = \{a, c\} \) constitutes a minimum vertex cover for the following graph:

```
   a---b---c---g
   |     |     |
   /     |     |
  /      |     |
 d      e    f
```

Formulate the minimum vertex cover problem as a Genetic Algorithm or another form of evolutionary optimization. You may use binary representation, OR any representation that you think is more appropriate. you should specify:

• A fitness function. Give 3 examples of individuals and their fitness values if you are solving the above example.

• A set of mutation and/or crossover and/or repair operators. Intelligent operators that are suitable for this particular domain will earn more credit.

• A termination criterion for the evolutionary optimization which insures that you terminate with a valid solution to the minimum vertex cover problem if possible without running indefinitely.