Machine Learning for Trading
Financial Investing

Capital Assets Pricing Model

• How the market impacts individual stocks
• Capital Asset Pricing Model (CAPM)
  – Equation.
• 1960s William Sharpe, Harry Markowitz, Merton Miller. Noble Prize 1990


• Portfolio – weighed set of assets.
• Example: Unleveraged Portfolio:
  • AAPL, GOOG, ORCL
  • 100% = [.6+.2+.2]
  • (Shorting is OK, then -.2)
    – Act of selling at a given price without possessing it, and buying it later at a lower price.
  – Portfolio:
    \[ \sum \text{abs}\left(w_i\right) = 1 \]
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– Return:
  \[ r_p(t) = \sum w_i r_i(t) = 1 \]
Example

- Stock A: 75% in portfolio +1% (up)
- Stock B: (-)25% in portfolio -2% (down)
- Return: 
  \[ \text{Return} = .75 * 1 + (-.25) = .75 + .50 = 1.25\% \]

Market Portfolio

Markets:
- US: S&P 500
- UK: FTA
- Japan: TOPIX

Indexes: Weighted.
Cap Weighted.

Market Cap – Capitalization
number of shares available for the stock times its price.

Outstanding shares -- stock currently held by all its shareholders/

Towards a CAPM

- Weight of any particular stock
  – Its market cap and divide it by the sum of the market caps of all the stocks.
  \[ \frac{\text{#shares} \times \text{price}}{\sum (\text{market caps of all stock})} \]
- Some Stocks have large weighing's, e.g., Apple and Exons each have about 5% of the S&P 500
  – And strong effect of the market.

- Return of an individual stock on day \( t \):
  – equals beta times the return on the market
  \[ r_i(t) = \beta \ r_m(t) + \alpha_i(t) \]
Compare Alpha and Beta

Consider these two scatter plots. This one is for stock XYZ versus S&P 500. This one is for stock ABC versus S&P 500. And each one of these dots represents the return for S&P 500 versus ABC or XYZ for that particular day.

Okay. So look at these two plots and tell me, which one has higher alpha and which one has higher beta?

So recall that beta is the slope of the line and alpha is where it hits the y-intercept there.

So, higher alpha. Well, ABC is intersecting the y-axis there much further up than XYZ so ABC’s got higher alpha.

In terms of beta, look, the slope of ABC is much higher than XYZ so ABC also wins on the higher beta question.

Recap.

- **Beta** reflects how risky an asset is compared to the overall market
  - A function of the volatility of the asset and the market (as well as the correlation between the two).
  - A risk-reward measure (risk worth in return for the reward)

- **Alpha** – measure of performance compared to the market.
  - `[ ]` for XYZ
  - `[ Y]` for ABC

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Alpha and Beta

\[ r_i(t) = \beta r_m(t) + \alpha_i(t) \]

- Higher Alpha? [ ] [ ]
- Higher Beta? [ ] [ ]

Alpha and Beta

\[ r_i(t) = \beta r_m(t) + \alpha_i(t) \]

- Higher Alpha? (Yint) [ ] [ ]
- Higher Beta? (slope) [ ] [ ]

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Alpha and Beta

\[ r_i(t) = \beta r_m(t) + \alpha_i(t) \]

- Higher Alpha? [ ] [ ]
- Higher Beta? [ ] [ Y]

Alpha and Beta

\[ r_i(t) = \beta r_m(t) + \alpha_i(t) \]

- Higher Alpha? [ ] [ ]
- Higher Beta [ Y] [ ]
Where would you invest?

- Safely deposit in bank
  - Fixed
    - 5% Interest

- Risky Company
  - Fixed
    - 5% Interest

Capital Asset Pricing Model - CAPM

- Expected return of an asset $\mathbf{r}_a$
- General Idea: Compensation comes in 2:
  1) **Time Value of Money**: Investment over time, and is typically represented by the Risk Free Rate $\mathbf{r}_f$ over a period of time
  2) **Risk Incurred**: Amount of compensation for taking additional risk.
    \[ \mathbf{r}_a = \mathbf{r}_f + \text{Risk Incurred} \]

What would sway you make a riskier investment?

- **Idea**: You want to be compensated for the added risk.
  - Want higher average percentage return.

- **Question**: What Average Percentage Return would compensates for the added risk?
  - What expected return of the investment is worth the added risk?

Capital Asset Pricing Formula.

Capital Asset Pricing Model - CAPM

1) **Time Value of Money**: Risk Free Rate $\mathbf{r}_f$ over a period of time
2) **Risk Incurred**: Amount of compensation for taking additional risk.

\[ \mathbf{r}_a = \mathbf{r}_f + \beta_a \times (\mathbf{r}_m - \mathbf{r}_f) \]
Going Back to Regression
How does it relate?

• \( r_a(t) = \beta_a r_m(t) + \alpha_a(t) \)

  \( \alpha_a(t) \), IS the residual - CAPM says this is random with an expected value of 0

• Lets assume the risk free rate is 0 (for now), in CAPM formula

  \( r_a(t) = \beta_a r_m(t) + \alpha_a(t) \)

• CAPM – significant return of an individual stock is due to the market.

• Now also imagine that we have a portfolio – with many different betas. Possibly moving in different directions.

\[ r_a(t) = \beta_a r_m(t) + \alpha_a(t) \]

\( \alpha_a(t) \), IS the residual - CAPM says this is random with an expected value of 0

Imagine many different stocks, many different betas, they may move in different directions.

\( r_a(t) = \beta_a r_m(t) + \alpha_a(t) \)

– Strategies:
  » Passive: Buy Index and HOLD
  » Active: Pick Stocks (believe in alpha – note \( \alpha \) is market relative).
  • Overweigh
  • Underweigh

Active Portfolio Construction

• \( r_p(t) = \beta_p r_m(t) + \alpha_p(t) \)

• \( r_a(t) = \beta_p r_m(t) + \sum w_a \alpha_a(t) \)

• Similarly \( \beta_p \) is weighted sum of the individual betas for each of the stocks.

Q: Implications of CAPM

<table>
<thead>
<tr>
<th></th>
<th>larger ( \beta_p )</th>
<th>smaller ( \beta_p )</th>
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<tbody>
<tr>
<td>Upward market</td>
<td>□</td>
<td>□</td>
</tr>
<tr>
<td>Downward market</td>
<td>□</td>
<td>□</td>
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</tbody>
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• If you have an upward market do you want a larger, or smaller \( \beta_p \)?
• If you have an downward market do you want a larger, or smaller \( \beta_p \)
• **Upward market** want a larger beta because then we go up even further than the market. So greater good to have greater than 1.

• **Downward market** — want smaller beta.
  — Example: if market goes down 1% and beta is less than 1, then our portfolio goes down less than the market, less than 1%.

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**Arbitrage Pricing Theory (ABT)**

• 1976 Stephen Ross.
• Don’t use a single Beta. Use different Beta per sectors e.g., different betas for Finance, Tech.

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**Implication of CAPM**

• \( r_p(t) = \beta_p r_m(t) + \alpha_p(t) \)
• Expected value of \( \alpha_p = 0 \)
• Only way to beat market is to choose \( \beta_p \)
• Choose high \( \beta_p \) in upward market
• Choose low \( \beta_p \) in downward market
• Efficient Market Hypothesis (EMH) says you cannot predict the market
  • Can you? What do you think?

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**CAPM and Hedge Funds.**

- **Two Stock Scenario**
  - over 10 days.
  - Assume Market is flat, did not move over time period
- A. Long $50.00
  - Predict stock is going up 1% over market
  - Beta = 1.0
- B. Short $50.00
  - Predict stock is going down -1% below market.
    • Negative BET
  - Beta = 2.0

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In flat market first term is 0:
Two Stock Scenario
- over 10 days.
- Assume Market goes down by 10% did not move over time period

A. Long $50.00
- Predict stock is going up 1% over market
- Beta = 1.0

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  - Negative BET
- Beta = 2.0

\[ r_a(t) = \beta_a \cdot r_m(t) + \alpha_a(t) \]

In flat market first term is 0:
For A our return is \(0.01 \cdot 50 = 0.50\)
For B our return is \((-1) \cdot -1 \cdot 0.01 = 0.01\)
Total is 1.00

Two Stock Scenario
- over 10 days.
- Assume Market goes up by 10%.

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- Beta = 2.0

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• Board Examples