

# Benford's Law

## Real World "Digit" Patterns



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## Today

- **Today: Benford's Law**
  - » HW: Applications of Benford's Law
- **Wednesday**
  - » Benford's Law Conclusion
- **Friday**
  - » Chapter 2 Discussion (Quiz?),
  - » Class Photo
- **HW: Summary Due Before class Friday**
  - » Chapter 2
  - » Hardcopy due: Friday before class
  - » Softcopy due midnight **before** the class Friday.

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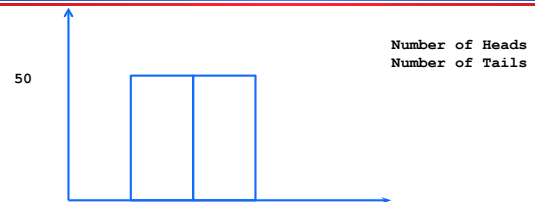
## Benford' Law is about Number Distributions

- Specifically about the Distribution of the First Digit in real world data.
- Before getting into the trenches of Benford's Law lets simply talk about (review) different Distributions.
  - » What do they look like?
- So lets explore:
  - » What is a numeric Distribution?

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## Oh No! Coin-toss Again

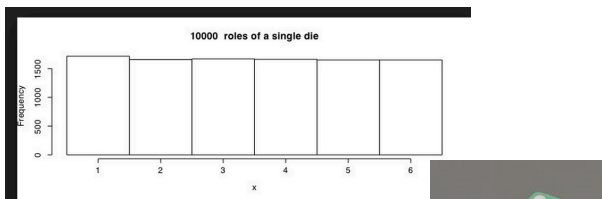


- **Uniform Distribution**
  - » Excel's Random : 0 to .99
  - =IF(INT(RAND()\*100)=1,"H","T")
  - INT: Rounds down to the nearest INT
  - Add .5 so half the numbers rounds down to 1 and half to zero.

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## 1000 Roles of a Single Die



- **Uniform Distribution**
  - =IF(INT(RAND()\*100)=1,"H","T")
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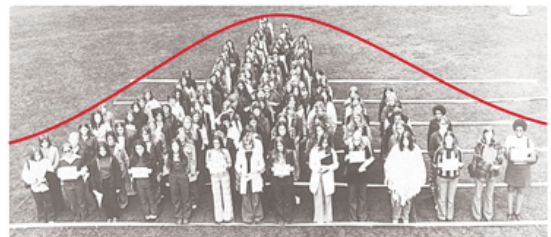


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## Example: Human Height

- IF we take the population of earth and then sort them according to height



- **Normal Distribution : Gaussian Distribution**

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## Bell Curve

- Heights of people
  - Size of things produced by machines
  - Blood pressure
  - Test Grades in a LARGE class.
- <http://www.mathsisfun.com/data/standard-normal-distribution.html>

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## What about Benford's Law? What is its Distribution?

Numeric Data	Leading Digit
235	2
975	9
124	1
536	5
611	6
385	3
196	1
745	7
385	3

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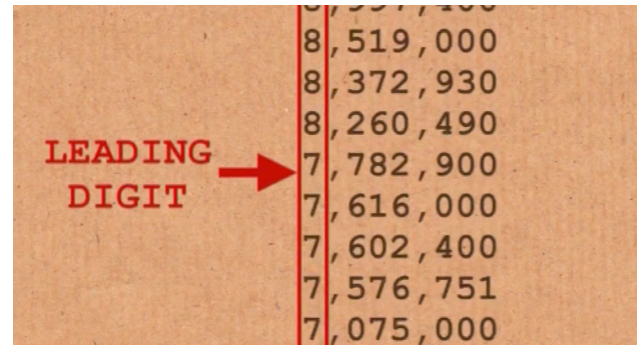
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## Example: Population of all the countries in the world



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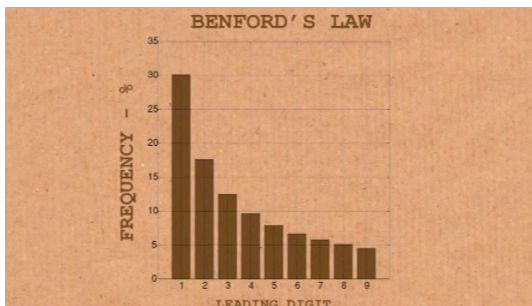
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## The Ideal: Benford's Law



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$d$	$P(d)$	Relative size of $P(d)$
1	30.1%	
2	17.6%	
3	12.5%	
4	9.7%	
5	7.9%	
6	6.7%	
7	5.8%	
8	5.1%	
9	4.6%	

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## When does it not work well?

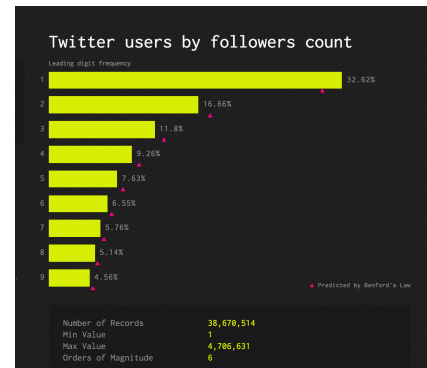
- Benford's Law works Accurately when:
  - » Numbers Span several order of magnitudes
- Example: Populations Goes from 10s to billions!
  - » Benford's Law applies YAY!

- When number's are assigned! Or there are thresholds (max/min).
  - » Check numbers
  - » Invoice numbers
  - » Number's with built in max and minimum

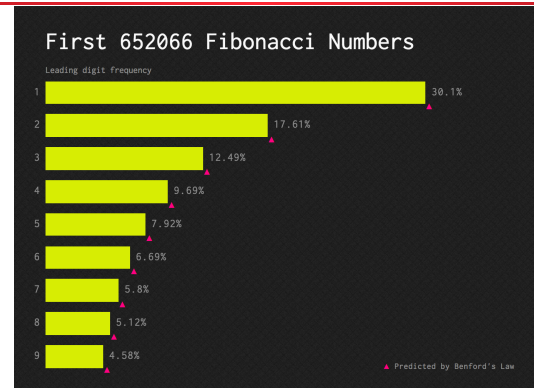
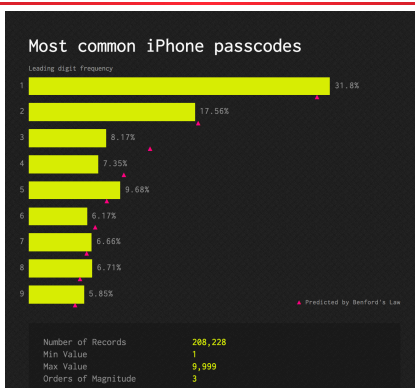
## Benford's Law

- Read Resource: Wikipedia!
- [http://en.wikipedia.org/wiki/Benford's\\_law](http://en.wikipedia.org/wiki/Benford's_law)

## More Examples



<http://testingbenfordslaw.com/twitter-users-by-followers-count>



## More Examples

- Heights of buildings
- Amount of money in your wallet
- Rolling of a die
  - » 1 Die, value of 'dots' have a uniform distribution
  - » 2 Dice: Question: Are Values of the dots:
    - Uniform
    - Non-Uniform
    - Benfordian

- <http://www.youtube.com/watch?v=O8N26edbqLM>

- 10 minute Videos:

- » <http://www.youtube.com/watch?v=vlSjbbADYs>
- » <http://www.youtube.com/watch?v=XXjIR2OK1kM>
  - (country from last video)