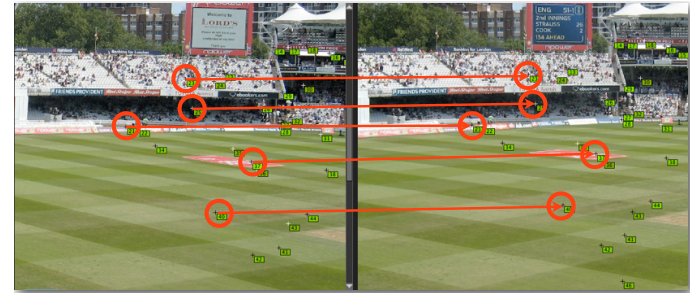


Combining Images

- Combining Images
 - Blending
 - Seam Carving
- Today: Use “similar” features

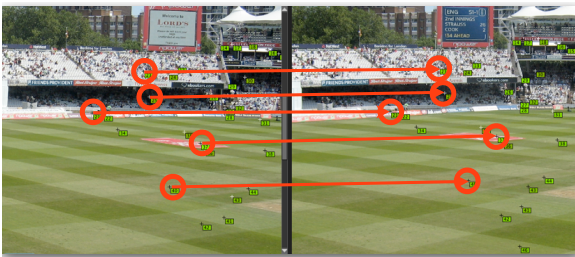


Courtesy: Irfan Essa

Corner Detection

Based on Richard Szeliski Notes
(textbook author)

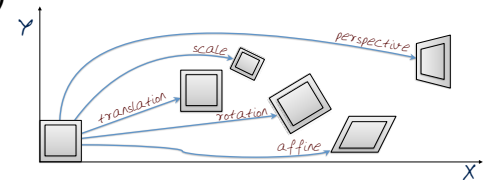
Challenge: Detect feature points in both Images



- Goal - Find points in an image that can be:
- Found in other images
- Found precisely - well localized
- Found reliably - well matched

What are Good Features?

- Distinctive
- Invariant to different acquisition conditions
 - Different view-points,
 - different illuminations,
 - different cameras (wide angle, different looking perspective)

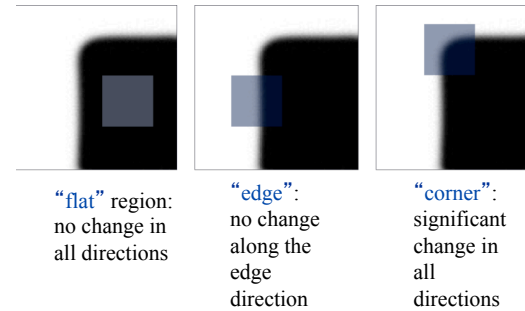


[Shi and C. Tomasi \(June 1994\). "Good Features to Track," \(citeseer\)](#)

Find Corners

- Key property: In the region around a corner, image gradient has
 - **two** or more dominant directions
- Corners are **repeatable** and **distinctive**

Corner Detection: Basic Idea

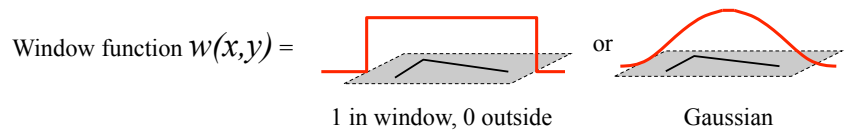
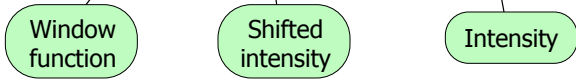


- Recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity

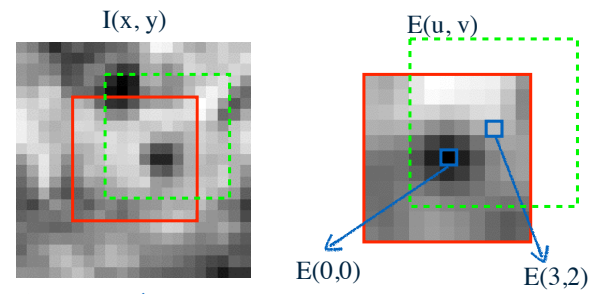
Basic Idea

Change of intensity for the shift $[u, v]$:

$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$



$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$



- Computation of the change in appearance by shifting the window by u, v :

Intuition

- Convert the above formula (uses Taylor Series expansion) –

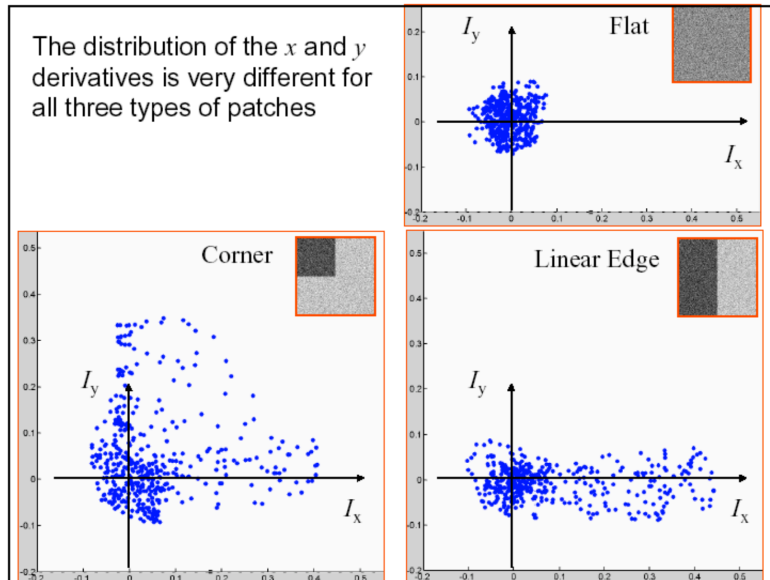
$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

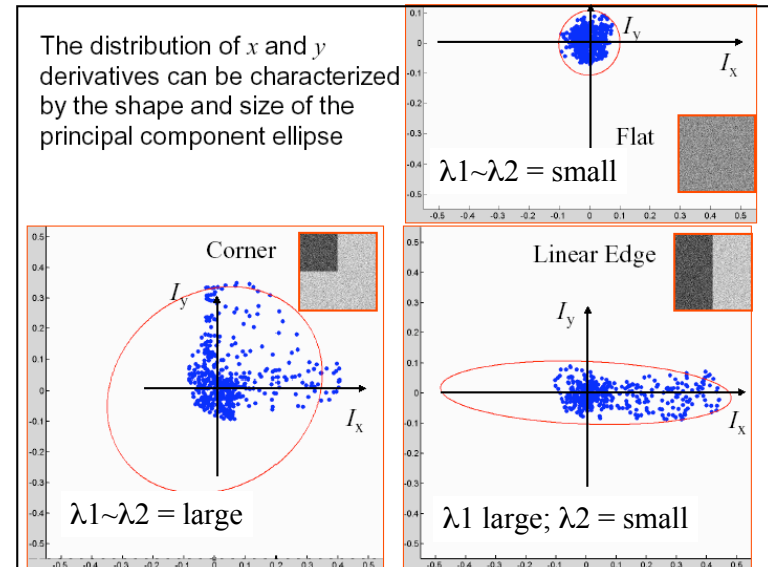
Familiar? These are just products of Component of the **gradients** I_x, I_y .

- M is matrix computed from image derivatives – M is also called structure tensor

- Treat gradient vectors as a set of (dx,dy) points with a center of mass defined as being at (0,0).
- Fit an ellipse to that set of points via scatter matrix
- Analyze ellipse parameters for varying case.



Courtesy: R. Collins, Penn State University

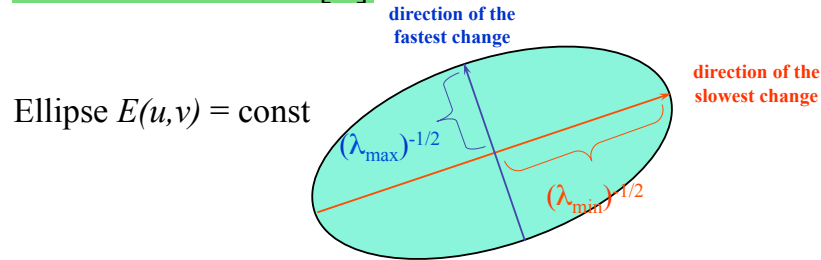


Courtesy: R. Collins, Penn State University

Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$E(u, v) \equiv [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } M$$



Harris Detector: Mathematics

Measure of corner response:

$$R = \det M - k (\text{trace } M)^2$$

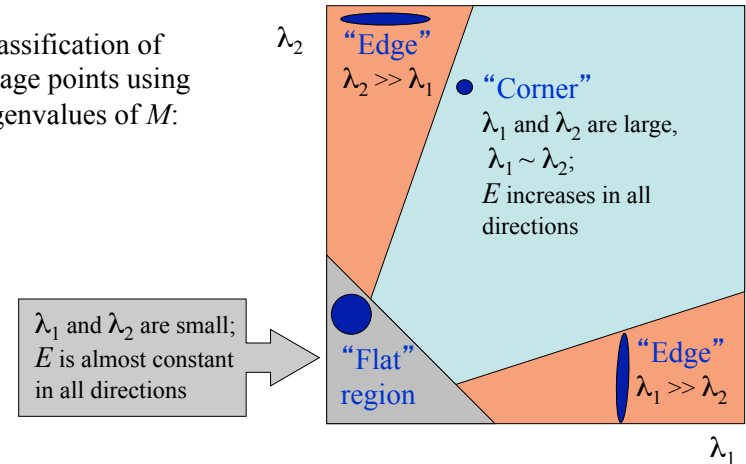
$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

(k – empirical constant, $k = 0.04-0.06$)

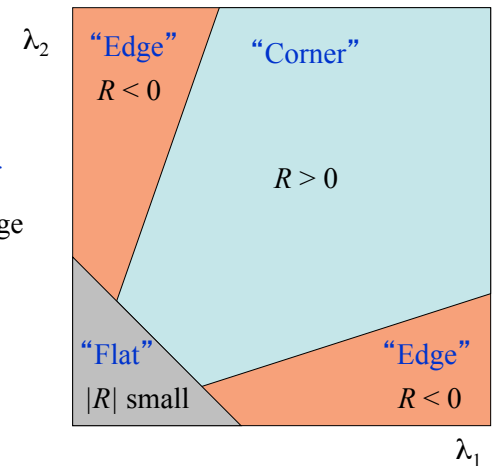
Harris Detector: Mathematics

Classification of image points using eigenvalues of M :



Harris Detector: Mathematics

- R depends only on eigenvalues of M
- R is large for a **corner**
- R is negative with large magnitude for an **edge**
- $|R|$ is small for a **flat** region



Harris Detector (Preview) 'Detect Corners'

- The Algorithm:
 - Compute Gaussian derivatives at each pixel
 - Compute matrix M in a ga
 - Find points with large corner response function R ($R > \text{threshold}$)
 - Take the points of local maxima of R

(we are not matching corner to 'another' image's corner Yet)

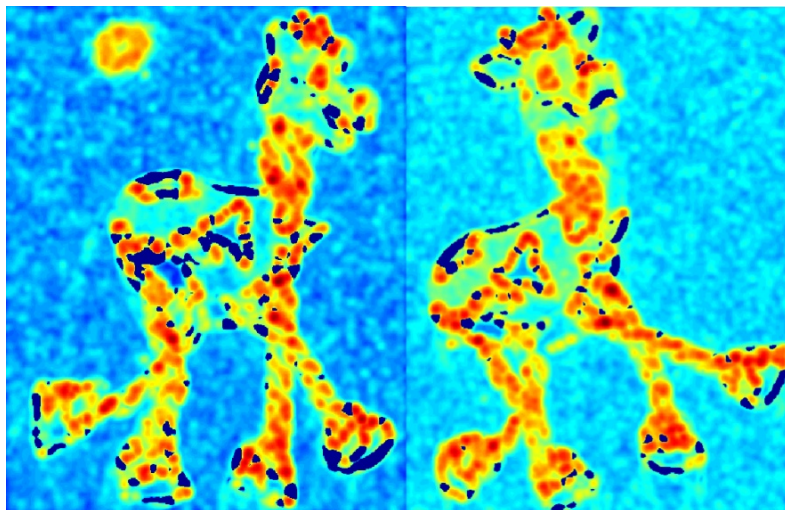
C. Harris and M. Stephens (1988). "A combined corner and edge detector"
Proceedings of the 4th Alvey Vision Conference. pp 147--151

Harris Detector: Workflow



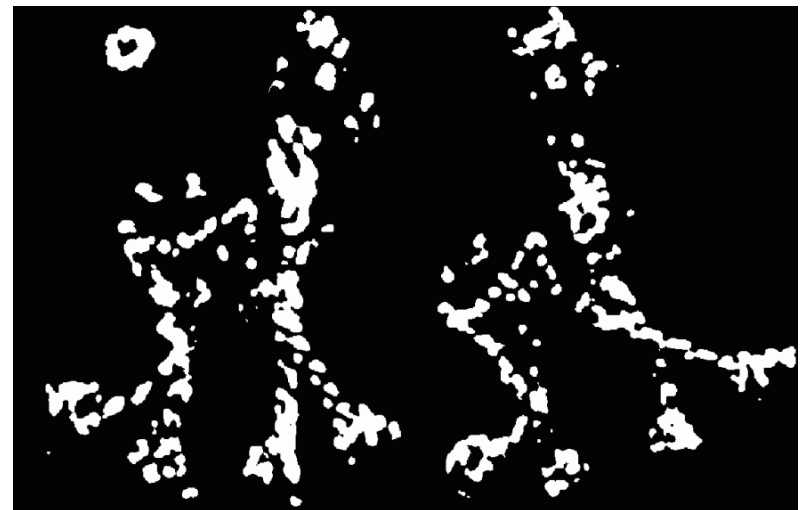
Harris Detector: Workflow

Compute corner response R



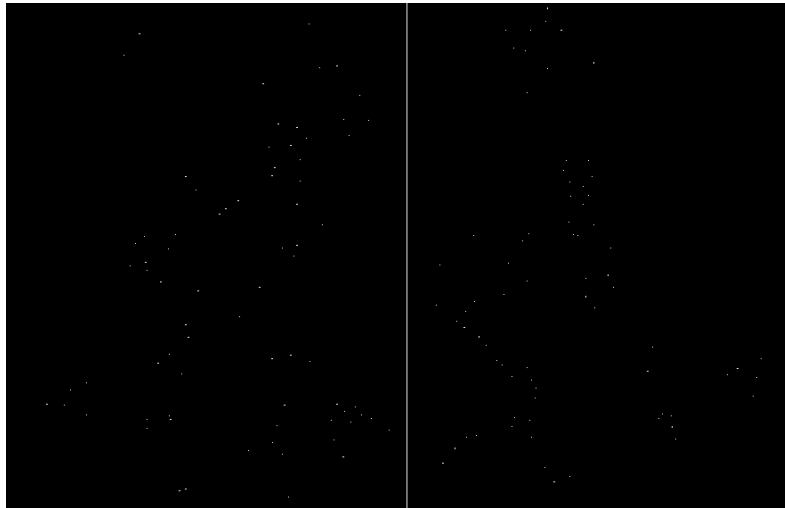
Harris Detector: Workflow

Find points with large corner response: $R > \text{threshold}$



Harris Detector: Workflow

Take only the points of local maxima of R



Harris Detector: Workflow



Harris Detector: Summary

- Average intensity change in direction $[u, v]$ can be expressed as a bilinear form:

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

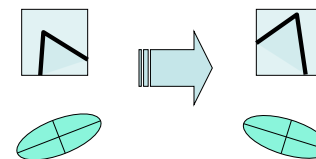
- Describe a point in terms of eigenvalues of M :
measure of corner response

$$R = \lambda_1 \lambda_2 - k (\lambda_1 + \lambda_2)^2$$

- A good (corner) point should have a *large intensity change in all directions*, i.e. R should be large positive

Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

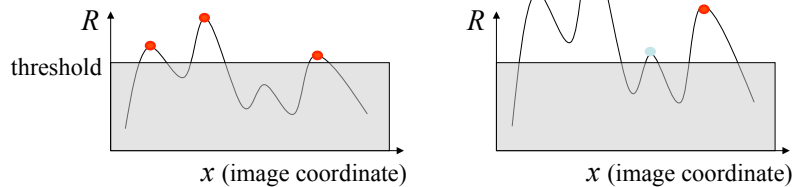
Corner response R is invariant to image rotation

Harris Detector: Some Properties

- Partial invariance to *affine intensity* change

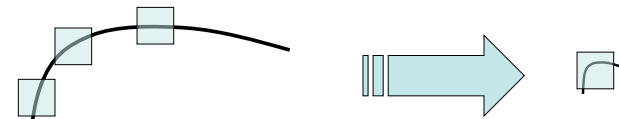
✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

✓ Intensity scale: $I \rightarrow a I$



Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as **edges**

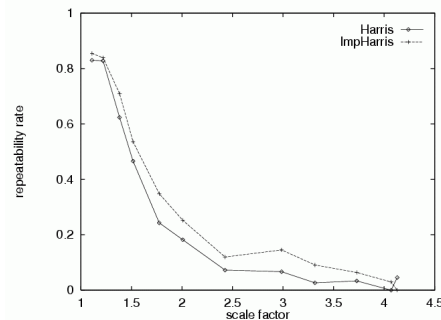
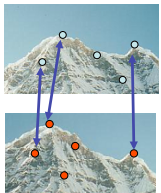
Corner !

Harris Detector: Some Properties

- Quality of Harris detector for different scale changes

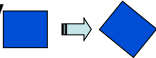
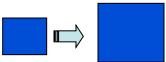


Repeatability rate:

$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



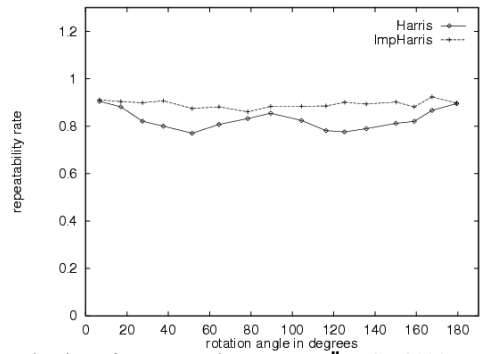
C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

Models of Image Change

- Geometry
 - Rotation 
 - Similarity (rotation + uniform scale) 
 - Affine (scale dependent on direction) valid for: orthographic camera, locally planar object 
- Photometry
 - Affine intensity change ($I \rightarrow a I + b$) 

Rotation Invariant Detection

- Harris Corner Detector



C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

Reading

https://en.wikipedia.org/wiki/Corner_detection

- See bibliography for additional resources

Shi and C. Tomasi (1994). Good Features to Track

- <http://citeseer.ist.psu.edu/viewdoc/summary?doi=10.1.1.135.7147>

Moravec (1980) Corner Detector (Ch 5-6) ([here](#)).

Harris & Stephens (1998), A Corner & Edge Detector

- <http://www.bmva.org/bmvc/1988/avc-88-023.pdf>

- Very readable.

Mikolajczyk and Schmid (2001). "Indexing Based on Scale Invariant Interest Points"

- <http://www.ee.surrey.ac.uk/CVSSP/Publications/papers/Mikolajczyk-ICCV-2001.pdf>

Lowe (2004) "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004

- <http://people.eecs.berkeley.edu/~malik/cs294/lowe-ijcv04.pdf>

Search for "Features" on OpenCV site