CSCI: 4500/6500 Programming Languages

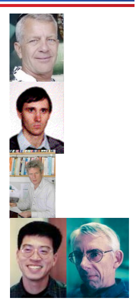
Prolog & Logic Programming

Thanks to: William W. Clocksin, Oxford University, UK.,Jason Eisner, John Hopkins University, James Lu & Jerud Mead, Bucknell University.

Prolog Download
Binaries and Source

- SWI-prolog (swipl 5.8.3) website:
  - » http://www.swi-prolog.org/
  - » Mac OS X on Intel & PPC (Tiger, Leopard (46.3 MB), Snow Leopard binaries available)
  - » Linux RPM
  - » Windows NT, XP, Vista, 2000
  - » Source Install
- XQuartz (X11) 2.5.0 for help & development tools.

Great Prolog Tutorials



- JR Fisher's original tutorial : http://www.csupomona.edu/~jrfisher/www/prolog_tutorial/contents.html
- Roman Barták's interactive tutorial: http://ktiml.mff.cuni.cz/~bartak/prolog/
- Mike Rosner's crash course: http://www.cs.um.edu.mt/~mros/prologcc/
- James Lu and Jerud Mead's tutorial: http://www.cse.ucsc.edu/classes/cmps112/Spring03/languages/prolog/PrologIntro.pdf
- James Power's tutorial: http://www.cs.nuim.ie/~jpower/Courses/PROLOG/ (2010 not available – let me know if you find it)

What is Prolog?

- Alain Colmeraeur & Philippe Roussel, 1971-1973
  - » With help from theorem proving folks such as Robert Kowalski
  - » Colmerauer & Roussel wrote 20 years later:

"Prolog is so simple that one has the sense that sooner or later someone had to discover it … that period of our lives remains one of the happiest in our memories.

What is Prolog?

- A *declarative* or *logic* programming language
  - » specifies the results (describes what the results look like)
    - – in contrast to a "procedure" on *how* to produce the results.
- Based on first order predicate calculus
  - » consists of propositions that may or may not be true
- Prolog uses logical variables
  - » Not the same as variables in other languages
  - » Used as 'holes' in data structures that are gradually filled in as the computation processes (will see examples)

Lets look at a sample session…

```
{saffron:ingrid:815} swipl
Welcome to SWI-Prolog (Multi-threaded, Version 5.6.9)
Copyright (c) 1990-2006 University of Amsterdam.
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software,
and you are welcome to redistribute it under certain conditions.
Please visit http://www.swi-prolog.org for details.

For help, use ?- help(Topic). or ?- apropos(Word).

?- ['second'].
% first compiled 0.00 sec, 596 bytes
```

```
repeat commands by traversing the command line
    history
CTRL-p
    moves up in command history
CTRL-n
    next command
<-  ->
    edit command line history
```

```
{saffron:ingrid:817} ls -l second.pl
-rw-r--r--    1 ingrid  ingrid  43 Apr 10 12:06 first.pl
{saffron:ingrid:818}
```

Look at a sample of code…

second.pl

```
elephant(kyle).     % this is a comment
elephant(kate).
panda(chi_chi).
panda(ming_ming).

dangerous(X) :- big_teeth(X).
dangerous(X) :- venomous(X).

guess(X,tiger) :- striped(X),big_teeth(X),isCat(X).
guess(X,koala) :- arboreal(X),sleepy(X).
guess(X,zebra) :- striped(X),isHorse(X).
```

Facts

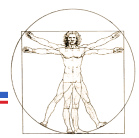
Rules

Prolog *Programs* are "Declarative"

I declare that the leaves are green and elephants are mammals.

- Clauses are statements about what is true about the problem (as statements and questions).
  - » instead of instructions on how to accomplish the solution.
- Prolog finds answers to queries by parsing through "the database" of possible solutions.

Anatomy of Prolog

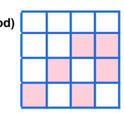


Declarative Component: "the program" ("the Database"):
- » Consists of facts and rules
- » Defines the relations on sets of values

Imperative Component : "the execution engine", the "Prolog Solver":
- » extracts the sets of data values *implicit* in the facts and rules of the program
- » Unification - matching query and "head" of rules (later)
- » Resolution - replaces the head with the body of the rule and then applies substitution to form a new query(ies).

Prolog as constraints programming

(Person, Food)



Person	Food
maria	olives
emmy	pear
eric	fish
isaac	chips
robert	fish
sean	chips

- Constraints between variables:
  Example: Person and Food.
- Facts:
  - » An identifier (name) of the constraint the followed by n-tuple of constants.
    - – Identifier (eats) names the relation
    - – the fact states that the tuple is in the relation
  - » Predicate: the relation identifier in combination with its parameters

```
eats(maria,olives).
eats(emmy,pear).
eats(eric,fish).
eats(isaac,chips).
eats(robert,fish).
eats(robert,chips).
```


Syntax of Terms

Term — UPPERCASE & _ DENOTES VARIABLES

Constant — *Names an individual*
- **Atom**: `flexible`, `spagetthi`, `super`, `califragilistic`, `expialidocious`, `+`, `=/=`, `'12Q&A'`
- **Number**: `0`, `1`, `57`, `1.618`, `2.04e-27`, `-13.6`

Compound Term — *Names an individual that has parts*

```
eats(maria, spaghetti)
book(dickens, Z, cricket)
f(x)
[1, 3, g(a), 7, 9]
-(+(15, 17), t)
15 + 17 - t
```

Variable — *Stands for an individual **unable** to be named when program is written*

```
X
Food
Poppins
_257
_
```

- Everything in prolog is built from *terms*:
  - » Prolog programs
  - » Data manipulated by Prolog programs

constant versus Variables

- Variables start with a capital letter, `A, B,…Z` or underscore `_`:
  - » `Food, Person, Person2, _A123`
- Constant "atoms" start with a, b, …z or appear in single quotes:
  - » `maria, olives, isaac, 'CSCI4500'`
  - » Other kinds of constants besides atoms:
    - – Integers `-7`, real numbers `3.14159`, the empty list `[]`
- Note: Atom is not a variable; it is not bound to anything, never equal to anything else

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

constant versus Variables

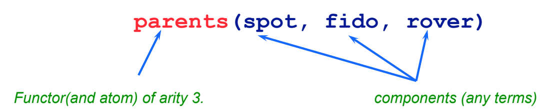
- Nothing stops you from putting constants into constraints:

```
% what Food does eric eat?
eats( eric, Food ).
% 2 answers: chips & pear
% use  ';' for next answer…

% what Person eats fish?
eats( Person, fish ).
% 2 answers: isaac & …?...

% who'll share what with robert? ** more later
eats(robert, Food), eats(Person, Food).
    Food=chips, Person=eric
```

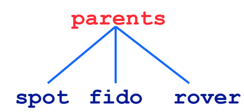
```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


`Familiar' Compound Terms

- The parents of Spot and Fido and Rover



- Can depict the *term* as a tree



Compound Terms

- An atom followed by a (parenthesized), comma-separated list of one or more terms: `x(y,z), +(1,2), .(1,[]), parent(adam,abel), x(Y,x(Y,Z))`
- A compound term can look like an SML, Scheme function call: `f(x,y)`
  - » Again, this is misleading
- Think of them as structured data

Summary Terms

```
<term>          ::= <constant> | <variable> | <compound-term>
<constant>      ::= <integer> | <real number> | <atom>
<compound-term> ::= <atom> ( <termlist> )
<termlist>      ::= <term> | <term> , <termlist>
```

- All Prolog programs and data are built from such terms
- Later, we will see that, for instance, `+(1,2)` is usually written as `1+2`
- But these are not new kinds of terms, just abbreviations

The Prolog Program (Database)

- A Prolog language system maintains a collection of facts and rules of inference
- It is like an internal database
- A Prolog program is just a set of data for this database
- The simplest kind of thing in the database is a *fact*: a term followed by a period

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


SWI-Prolog

```
{atlas:maria:141} swipl
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.3)
Copyright (c) 1990-2003 University of Amsterdam.
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software,
and you are welcome to redistribute it under certain conditions.
Please visit http://www.swi-prolog.org for details.

For help, use ?- help(Topic). or ?- apropos(Word).

?-
```

- Prompting for a query with `?-`
- Normally interactive: get query, print result, repeat

The `consult` Predicate

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

```
?- consult(eats).
% eats compiled 0.00 sec, 0 bytes

true.
?- [eats].

% eats compiled 0.00 sec, 0 bytes

false.
```

- Predefined predicate to read a program from a file into the database
  - » Example: File `eats.pl` defines the "eats" constraints, or lists of facts.

Simple Queries


```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


- A query asks the language to prove something
- The answer will be True or False
- Some queries, like `consult` are executed only for their side effects.
- Example Query program:
  - » Does `kyle` eat `fish` (type query)?

Here constraints acts as a procedure or function →

```
?- eats(adam,sushi).
true.
?- eats(jordan,vegetables).
false.
```


Simple Queries: the Period '.'


```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


- Queries can take multiple lines
- If you forget the final period, Prolog prompts for more inputs with |.

```
?- eats(ibti,vegetables)
```

No period

Simple Queries: the Period '.'


```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


- Queries can take multiple lines
- If you forget the final period, Prolog prompts for more inputs with `|`.

```
?- eats(ibti,vegetables)
|
```

Prolog prompt

curser

Simple Queries: the Period '.'


```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```


- Queries can take multiple lines
- If you forget the final period, Prolog prompts for more inputs with |.

```
?- eats(ibti,vegetables)
|    .

false.
```


Queries With Variables

```
?- eats(michael,X).

X = fish

true.
?-
```

Here, it waits for input. We hit Enter (or ;) to make it proceed.

- Any term can appear as a query, including a term with variables
- The Prolog system shows the bindings necessary to prove the query

Multiple Solutions

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

- There might be more than one way to prove the query
- By typing `;` rather than Enter, you ask the Prolog system to find more solutions
  - » Example: What does `kyle` eat?

";" (no return) Asks: anymore values that satisfy the query?

```
?- eats(isaac,X).
X = fish ;
X = chips ;
No
```


Flexibility

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

- Normally, variables can appear in any or all positions in a query:
  - » `eats(X,olives)`
  - » `eats(corey,X)`
  - » `eats(X,Y)`
  - » `eats(X,X)`
    - – (guesses)?

Conjunctions

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

```
% who'll share what with eric?

?- eats(eric, Food), eats(Person, Food).
Food = chips
Person = eric;

Food = chips
Person = isaac;
```

- A conjunctive query has a list of query terms separated by commas
  - » think of commas as "AND's"
- The Prolog system tries prove them all (using a single set of bindings)
- **Example:** Query folks that eat *common* foods with `eric`

More General Queries

```
eats(adam, sushi).
eats(eric,chips).
eats(eric,pears).
eats(isaac,fish).
eats(isaac,fish).
east(ibti,chips).
east(ibti, sushi).
eats(jordan,fish).
eats(jordan,olives).
eats(jonathan,olives).
eats(jonathan,chips).
eats(maria, sushi).
eats(robert,chips).
eats(robert,olives).
eats(sean, sushi).
eats(sean,chips).
eats(young,olives).
eats(young,pears).
```

- Query folks that eat *common* foods:
  - » conjoin two constraints with a common food.
  - » conjoined with a comma (read as "and').

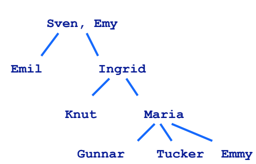
```
?- eats(Person1,Food),eats(Person2,Food).
```

Both Adam and Maria like sushi

```
Person1 = adam
Food = sushi
Person2 = adam;

Person1 = adam
Food = sushi
Person2 = maria;
```


More Examples: Conjunctions

```
Who are Sven's grandchildren?
%
% 1) Who is a child of Sven?
%        Assume 'Child'
% 2) Who is a child of Child?
%        Assume 'GrandChild'
?- parent(sven,Child),
|    parent(Child, GrandChild).

Child = ingrid,
GrandChild = maria ;

Child = ingrid,
GrandChild = knut ;

No
?-
```

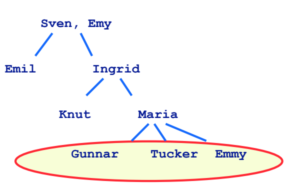


mariafamily.pl

```
parent(maria,gunnar).
parent(maria,tucker).
parent(maria,emmy).
parent(ingrid,maria).
parent(ingrid,knut).
parent(emy,ingrid).
parent(sven,ingrid).
parent(sven,emil).
```


More Examples: Conjunctions

```
Great grandchildren of Emy?
% Great grandchildren of Emy?
% 1) Who is a child of Emy
% 2) Who is a child of ?
% 3) Who is a child of ?
```

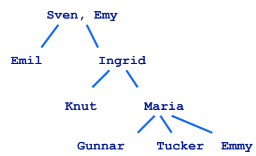



```
parent(maria,gunnar).
parent(maria,tucker).
parent(maria,emmy).
parent(ingrid,maria).
parent(ingrid,knut).
parent(emy,ingrid).
parent(sven,ingrid).
parent(sven,emil).
```

More Examples: Conjunctions

Great grandchildren of Emy?

```
% Great grandchildren of Emy?

?- parent(emy,Child),
|    parent(Child,Grandchild),
|    parent(Grandchild,GreatGrandchild).

Child = ingrid
Grandchild = maria
GreatGrandchild = gunnar ;

Child = ingrid
Grandchild = maria
GreatGrandchild = tucker ;

Child = ingrid
Grandchild = maria
GreatGrandchild = emmy ;

No
?-
```




```
parent(maria,gunnar).
parent(maria,tucker).
parent(maria,emmy).
parent(ingrid,maria).
parent(ingrid,knut).
parent(emy,ingrid).
parent(sven,ingrid).
parent(sven,emil).
```


Motivation: Rules

parents.pl

```
% Great grandchildren of Emy?

?- parent(emy,Child),
|    parent(Child,Grandchild),
|    parent(Grandchild,GreatGrandchild).
```

- Long query for great grandchildren of Emy?
  - » Nicer to query directly:
    `greatgrandparent(emy, GreatGrandchild)`
  - » While not adding separate facts of that form to the database?
    - – this relation should follow from the parent relation already defined.

A Rule

head →

```
greatgrandparent(GGP,GGC) :-
    parent(GGP,GP),
    parent(GP,P),
    parent(P,GGC).
```

- A rule says *how to prove something*: to prove the *head*, prove its *conditions*
- To prove `greatgrandparent(GGP,GGC)`, find some `GP` and `P` for which you can prove `parent(GGP,GP)`, then `parent(GP,P)` and then finally `parent(P,GGC)`

A Rule

head →

```
greatgrandparent(GGP,GGC) :-
    parent(GGP,GP),
    parent(GP,P),
    parent(P,GGC).
```

- A rule says how to prove something: to prove the head, prove the conditions
- To prove `greatgrandparent(GGP,GGC)`, find some `GP` and `P` for which you can prove `parent(GGP,GP)`, then `parent(GP,P)` and then finally `parent(P,GGC)`

A Rule

head →

```
greatgrandparent(GGP,GGC) :-
    parent(GGP,GP),
    parent(GP,P),
    parent(P,GGC).
```

← *conditions (body)*

- A rule says how to prove something: to prove the head, prove the conditions
- To prove `greatgrandparent(GGP,GGC)`, find some `GP` and `P` for which you can prove `parent(GGP,GP)`, then `parent(GP,P)` and then finally `parent(P,GGC)`

Facts and Rules

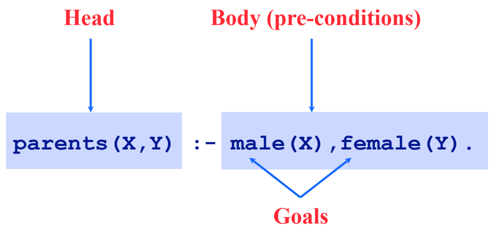
"if" body is true
"provided that"
"turnstile"
- it's supposed to look like "←"

```
Head :- Body.   % This is a rule.
Head.           % This is a fact.
```

Head is the **consequence**.

Head can be concluded if the body is true

Facts and Rules



- Note that left side of the rule looks just like a fact, except that the parameters are variables
- Read:
  - » The pair “parents(X,Y)” satisfies the predicate “parents” if there is a node X and Y such that X satisfies the predicate “X” and “Y” satisfies the predicate Y.

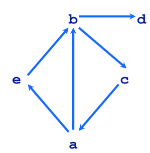
Clauses

- A program consists of a list of *clauses*
- A clause is either a fact or a rule, and ends with a period

```
parent(maria,gunnar).
parent(maria,tucker).
parent(maria,emmy).
parent(ingrid,maria).
parent(ingrid,knut).
parent(emy,ingrid).
parent(sven,ingrid).
parent(sven,emil).
greatgrandparent(GGP,,GGC) :-
        parent(GGP,GP),
        parent(GP,P),
        parent(P,GGC).
```


Example: Clauses: Facts and Rules

- Example: A directed graph of five nodes:



- Define the edges of the graph, as facts?
- Define a rule called “tedge” which defines the property of a “path of length two” between two edges?

```
edge(a,b).
edge(a,e).
edge(b,d).
edge(b,c).
edge(c,a).
edge(e,b).
```

```
tedge(Node1,Node2) :-
    edge(Node1,SomeNode),

        edge(SomeNode,Node2).
```

The pair (Node1,Node2) satisfies the predicate tedge if there is a node SomeNode such that the pairs (Node1,SomeNode) and (SomeNode,Node2) both satisfies the predicate edge.

Interpretation of Clauses

- Form of Clause:
  - » H :- $G_1, G_2, \ldots, G_n$.
- Declarative Reading:
  - » “That H is provable follows from goals $G_1, G_2, \ldots, G_n$ being provable”
- Procedural Reading:
  - » “To execute procedure H, the procedures called by the goals $G_1, G_2, \ldots, G_n$ are executed first”

Example 3: Another Rule

```
Compatible(Person1, Person2) :- eats(Person1,Food),
    eats(Person2,Food).
```

- “Person1 and Person2 are compatible if there exists some Food that they both eat.”
- “One way to satisfy the head of this rule is to satisfy the body

```
eats(steve,olives).
eats(sol,pear).
eats(sol,fish).
eats(george,chips).
eats(cole,fish).
eats(cole,chips).
eats(alex,olives).
eats(corey,olives).
eats(george,olives).
eats(jason,olives).
eats(dong,olives).
eats(david,olives).
```


Rules using ‘other’ Rules

```
grandparent(GP,GC) :-
  parent(GP,P), parent(P,GC).

greatgrandparent(GGP,GGC) :-
  grandparent(GGP,P), parent(P,GGC).
```

- Same relation, defined indirectly
- Note that both clauses use a variable `P`
- The scope of the definition of a variable is the clause that contains it

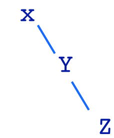
Prolog allows recursion SQL doesn’t

Recursive Rules

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :-
           parent(Z,Y),
           ancestor(X,Z).
```

- X is an ancestor of Y if:
  - Base case: X is a parent of Y
  - Recursive case: there is some Z such that Z is a parent of Y, and X is an ancestor of Z
- Prolog tries rules in the order given, so put base-case rules and facts first

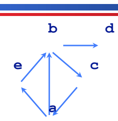
Recursion Example 2



- Who's married to their boss?
  - boss(X,Y), married(X,Y).
- Who's married to their boss's boss?
  - boss(X,Y), boss(Y,Z), married(X,Z).
- Who's married to their boss's boss's boss?
  - Okay, this is getting silly. Let's do the general case.
- Who's married to someone *above* them?
  - above(X,X).
  - above(X,Y) :- boss(X,Underling), above(Underling,Y).
  - above(X,Y), married(X,Y).

Base case: For simplicity, it says that X is "above" herself. If you don't like that, replace base case with above(X,Y) :- boss(X,Y).

Example: Graph Example



- Embellish graph program to include "path"s of any positive length.
- Thinking Recursively:
  - If there is an edge then there is a path (base)
  - If there is an edge to an intermediate node from which there is a path to the final node.

```
path(N1,N2)        :- edge(N1,N2).
path(N1,N2)        :- edge(N1,SomeN),path(SomeN,N2)
```

  - Two rules with the same head, reflects logical "or"
  - Predicate of head of second rule, is also in the body of that rule.
  - These rules together illustrate recursion in Prolog!

```
edge(a,b).                 edge(b,c).
edge(a,e).                 edge(c,a).
edge(b,d).                 edge(e,b).
tedge(N1,N2)               :- edge(N1,SomeN),edge(SomeN,N2).
path(N1,N2)                :- edge(N1,N2).
path(N1,N2)                :- edge(N1,SomeN),path(SomeN,N2).
```


Core Syntax of Prolog

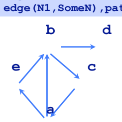
```
<clause>    ::=  <fact> | <rule>
<fact>      ::=  <term> .
<rule>      ::=  <term> :- <termlist> .
<termlist>  ::=  <term> | <term> ,
<termlist>
```

- You have seen the complete core syntax
- There is not much more syntax for Prolog than this: it is a very simple language
- Syntactically, that is!

How does Prolog Compute?

- Deduce useful implicit knowledge from the "program" or data base.
- Computations in Prolog is facilitated by the *query*, a conjunction of atoms.
- New example (more complicated) program:

```
1  edge(a,b)                 edge(b,c)
   edge(a,e).                edge(c,a).
   edge(b,d).                edge(e,b).
   tedge(N1,N2)              :- edge(N1,SomeN),edge(SomeN,N2).
2  path(N1,N2)               :- edge(N1,N2).
   path(N1,N2)               :- edge(N1,SomeN),path(SomeN,N2)
```




```
1 edge(a,b).                2 edge(b,c).
3 edge(a,e).                4 edge(c,a).
5 edge(b,d).                6 edge(e,b).
7 tedge(N1,N2)                :- edge(N1,SomeN),edge(SomeN,N2).
8 path(N1,N2)                 :- edge(N1,N2).
9 path(N1,N2)                 :- edge(N1,SomeN),path(SomeN,N2)
```

- edge(a,b).

```
1 edge(a,b).            2 edge(b,c)
3 edge(a,e).            4 edge(c,a).
5 edge(b,d).            6 edge(e,b).
6 tedge(N1,N2)            :- edge(N1,SomeN),edge(SomeN,N2).
7 path(N1,N2)             :- edge(N1,N2).
8 path(N1,N2)             :- edge(N1,SomeN),path(SomeN,N2)
```

- `edge(a,b).`
  - » Iterates in order through the program's "`edge`" clauses.
  - » *Ground Query* only value identifiers as parameters to the predicate.
  - » First one to match is `edge(a,b).` so Prolog returns with `true` (so yes).

```
1 edge(a,b).            2 edge(b,c)
3 edge(a,e).            4 edge(c,a).
5 edge(b,d).            6 edge(e,b).
6 tedge(N1,N2)            :- edge(N1,SomeN),edge(SomeN,N2).
7 path(N1,N2)             :- edge(N1,N2).
8 path(N1,N2)             :- edge(N1,SomeN),path(SomeN,N2)
```

- `edge(a,b).`
- `path(a,b).`

```
1 edge(a,b).            2 edge(b,c)
3 edge(a,e).            4 edge(c,a).
5 edge(b,d).            6 edge(e,b).
7 tedge(N1,N2)            :- edge(N1,SomeN),edge(SomeN,N2).
8 path(N1,N2)             :- edge(N1,N2).
9 path(N1,N2)             :- edge(N1,SomeN),path(SomeN,N2)
```

- `edge(a,b).`
- `path(a,b).`
  - » another ground query
  - » No rule that exactly match the query.
  - » Know, the head is true if the body is true
  - » If variable's `N1` and `N2` are replaced by `a` and `b`, then body of 8 is true
    - – `edge(a,b)` is a fact!
    - – and the head with the same substitution must be true
  - » Prolog conclude that the query is true

```
1 edge(a,b).            2 edge(b,c)
3 edge(a,e).            4 edge(c,a).
5 edge(b,d).            6 edge(e,b).
7 tedge(N1,N2)            :- edge(N1,SomeN),edge(SomeN,N2).
8 path(N1,N2)             :- edge(N1,N2).
9 path(N1,N2)             :- edge(N1,SomeN),path(SomeN,N2)
```

- `edge(a,b).`
- `path(a,b).`
- `tedge(a,X).`

```
1 edge(a,b).            2 edge(b,c)
3 edge(a,e).            4 edge(c,a).
5 edge(b,d).            6 edge(e,b).
7 tedge(N1,N2)            :- edge(N1,SomeN),edge(SomeN,N2).
8 path(N1,N2)             :- edge(N1,N2).
9 path(N1,N2)             :- edge(N1,SomeN),path(SomeN,N2)
```

- `edge(a,b).`
- `path(a,b).`
- `tedge(a,X).`
  - » non-Ground Query: variable parameters
  - » Scan rules, finds that constraint '7' defines `tedge`, focus on 7
  - » Substitutes `N1 = a`, `X = N2`
  - » Is edge(a, N2) true? True if body is true, evaluates body:
    - » `edge(a,SomeN), edge(SomeN,N2)`?
  - » `edge(a,SomeN)`? two facts fit, take the first one edge(a,b)
    - » if we substitute SomeN = b [first query is satisfied]
  - » after substitution evaluate 2nd atom, i.e. `edge(b,N2)`?
  - » Similarly as above substitute: `N2 = d`
  - » Following the substitution it finds that `X = d` satisfies the original query

How Does Prolog Compute?

- Unification (pattern matching, eval).
- Resolution (apply, one at a time).
- Backtracking

Unification

- Pattern-matching using Prolog terms
- Two terms unify if there is some way of binding their variables that make them identical.
  - » Usually the two terms
    - – one from the query (or another goal) and
    - – the other being a *fact* or a *head of a rule*
  - » Example:
    - – `parent(adam,Child)` and `parent(adam,seth)`
    - – Do these unify?
    - – Yes! they unify by binding the variable `Child` to the atom `seth`.

Resolution

- The hardwired inference step
- A clause is represented as a list of terms (a list of one term, if it is a fact)
- Resolution step applies one clause, once, to make progress on a list of goal terms

Resolution

- When an atom from the query has unified with the head of of a rule (or a fact),
- Resolution replaces the atom with the body of the rule (or nothing, if a fact) and
- then applies the substitution to the new query.

tedge(a,X).

```
1 edge(a,b)            edge(b,c)
3 edge(a,e).           edge(c,a).
5 edge(b,d).           edge(e,b).
6 tedge(N1,N2)         :- edge(N1,SomeN),edge(SomeN,N2).
7 path(N1,N2)          :- edge(N1,N2).
8 path(N1,N2)          :- edge(N1,SomeN),path(SomeN,N2)
```

- Unify:
  - » tedge(a,X) and tedge(N1,N2).
  - » giving the substitution
    - – N1 =a, X = N2
- Resolution:
  - » replaces tedge(a,X) with body edge(N1,SomeN), edge(SomeN,N2) and apply the substitution above to get the new query.
    - edge(a,SomeN),edge(SomeN,N2)
- Select first atom, edge(a,SomeN)
- Unify:
  - » edge(a,SomeN) with edge(a,b),
  - » giving the substitution
    - – SomeN = b
- Resolution: replace edge(a,SomeN) …

tedge(a,X).

```
1 edge(a,b).           2 edge(b,c)
3 edge(a,e).           4 edge(c,a).
5 edge(b,d).           6 edge(e,b).
6 tedge(N1,N2)           :- edge(N1,SomeN),edge(SomeN,N2).
7 path(N1,N2)            :- edge(N1,N2).
8 path(N1,N2)            :- edge(N1,SomeN),path(SomeN,N2)
```

- Resolution: replace edge(a,SomeN) by nothing (since we unified with a fact) and apply the substitution above to get the new query:
  - » edge(b,N2)
- There is only one atom in the query.
- Unify
  - » edge(b,N2), and edge(b,d).
- giving the substitution
  - » N2 = d
- Resolution: replace edge(b,N2) by nothing (since we unified with a fact). Since the resulting query is empty we are done!

____Backtracking

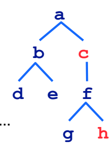
- There are other solutions, we could redo the computation above and get substitution
  - » X=b or X = c or X =d
- When Prolog reduces a query to the empty query,
  - » it backtracks to the most recent unification to determine whether there is another fact or rule with which the unification can succeed.
  - » Backtracking continues until all possible answers are determined.

Recursive Queries

```
above(X,X).
above(X,Y) :- boss(X,Underling), above(Underling,Y).
```

- above(c,h). % should return True
  - » matches above(X,X)? no
  - » matches above(X,Y) with X=c and Y=h
  - » boss(c,Underling),
    - – matches boss(c,f) with Underling=f
  - » above(f,h).
    - – matches above(X,X)? no
    - – matches above(X,Y) with X=f, Y=h
      - boss(f,Underling),
        - » matches boss(f,g) with Underling=g
      - above(g,h)
        - » … ultimately fails because g has no underlings…

```
boss(a,b).  boss(a,c).
boss(b,d).  boss(c,f).
boss(b,e).  boss(f,g).
boss(f,h).
```

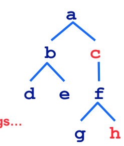


Recursive Queries

```
above(X,X).
above(X,Y) :- boss(X,Underling), above(Underling,Y).
```

- above(c,h). % should return True
  - » matches above(X,X)? no
  - » matches above(X,Y) with X=c and Y=h
  - » boss(c,Underling),
    - – matches boss(c,f) with Underling=f
  - » above(f,h).
    - – matches above(X,X)? no
    - – matches above(X,Y) with X=f, Y=h
      - boss(f,Underling),
        - » matches boss(f,g) with Underling=g
      - above(g,h)
        - » … ultimately fails because g has no underlings…

```
boss(a,b).  boss(a,c).
boss(b,d).  boss(c,f).
boss(b,e).  boss(f,g).
boss(f,h).
```

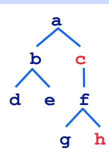


Recursive Queries

```
above(X,X).
above(X,Y) :- boss(X,Underling), above(Underling,Y).
```

- above(c,h). % should return True
  - » matches above(X,X)? no
  - » matches above(X,Y) with X=c and Y=h
  - » boss(c,Underling),
    - – matches boss(c,f) with Underling=f
  - » above(f,h).
    - – matches above(X,X)? no
    - – matches above(X,Y) with X=f, Y=h
      - boss(f,Underling),
        - » matches boss(f,Underling) with Underling=h
      - above(h,h)
        - » matches above(X,X) with X=h …

```
boss(a,b).  boss(a,c).
boss(b,d).  boss(c,f).
boss(b,e).  boss(f,g).
boss(f,h).
```



Review: Basic Elements of Prolog

- **Variable: any string of letters, digits, and underscores beginning with an Uppercase letter**
- **Instantiation: binding of a variable to a value**
  - » Lasts only as long as it takes to satisfy one complete goal
  - » allows unification to succeed
- **Predicates: represents atomic proposition**

  functor(parameter list)

Review Prolog

- **Prolog program: Set of propositions**
  - » Facts
  - » Rules: consequence ⇐ antecedent (if antecedent is true then the consequence is true).
    - – `edge(A,B) :- edge(A,X),edge(X,B).`
- **Running a program: A Prolog query (sometimes called goals): A proposition of which truth is to be determined.**
  - » Idea: Prove truthfulness (or "cannot determine" (not falsehood)) by trying to find a chain of inference rules and facts (inference process)
    - Resolution: Process that allows inferred propositions to be computed from given propositions
  - » Unification merges compatible statements. Binding process.

Inference Process

- **Backward Chaining, Top-down resolution:**
  - » Start with goal (query), see if a sequence of propositions leads to set of facts in the database (Prolog)
    - – Looks for something in the database that unify the current goal,
      - finds a fact, great it succeeds!
      - If it finds a rule, it attempts to satisfy the terms in the body of the rule (these are now subgoals).
- **Forward Chaining, Bottom-up resolution:**
  - » Begin with program of facts and rules in the database and attempt to find a sequence that leads to goal (query).

Backward Chaining

- When goal has more than one sub-goal, can use either
  - » Depth-first search: find a complete proof for the first sub-goal before working on others (Prolog)
    - – Push the current goal onto a stack,
    - – make the first term in the body the current goal, and
    - – prove this new goal by looking at beginning of database again.
    - – If it proves this new goal of a body successfully, go to the next goal in the body. If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.
  - » Breadth-first search: work on all sub-goals in parallel

Backtracking

- If a sub-goal fails:
  - » reconsider previous subgoal to find an alternative solution
- Begin search where previous search left off
- Can take lots of time and space because may find all possible proofs to every sub-goal

Compound Terms

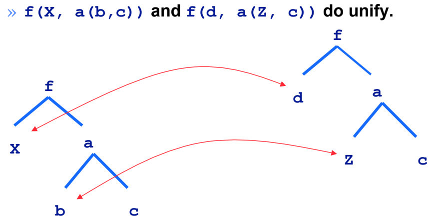
- Basic blocks: variables, constants and variables
- Compound terms: Seen it already -- it is the `functor( parameter list )` structure (e.g., `eats( cole,fish )`)
  - » Variables cannot be used for the `functor`
  - » However the "parameter list" can be any kind of term (it can be another functor).
  - » `book( title(lord_of_the_rings), author(tolkien) )`
  - » Uh uh what about unification now! (matching of goals and heads).

Unification Rules

- Two terms unify:
  - » if substitution can be made for any variables in the terms so that terms are made identical.
  - » If no such substitution exists, the terms do not unify.
- The unification algorithm proceeds by recursively descent of the two terms.
  - » Constants unify if they are identical
  - » Variables unify with any term, including other variables
  - » Compound terms unify if their functors and components unify

Unification Compound Terms

- Compound terms unify if their functors and components unify (how do terms become equal?)
  - » `f(X, a(b,c))` and `f(d, a(Z, c))` do unify.

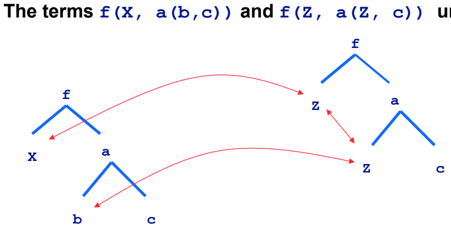


These terms are made equal if d is substituted for X, and b is substituted for Z.

- » d is substituted for X (X is instantiated to d, X/d)
- » b is substituted for Z (Z is instantiated to b, Z/b)

Example 2

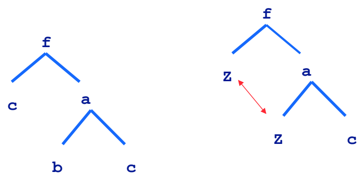
- The terms `f(X, a(b,c))` and `f(Z, a(Z, c))` unify



- `Z` co-refers within the term. Here, `X/b`, `Z/b`.
  - »`Earlier :f(X, a(b,c))` and `f(d, a(Z, c))` did unify…

What about?

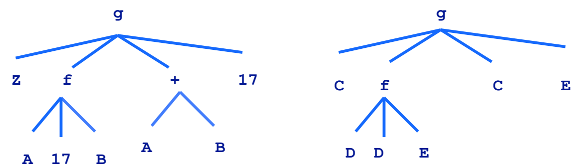
- `f(c, a(b,c))` and `f(Z, a(Z, c))` ?



- No matter how hard you try, these terms cannot be made identical by substituting terms for variables.

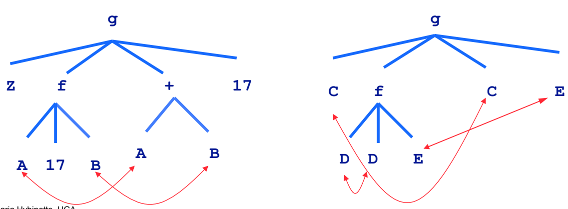
Unify?

- `g(Z,f(A,17,B),A+B,17)` and
- `g(C, f(D, D, E), C, E)` ?



Unify?

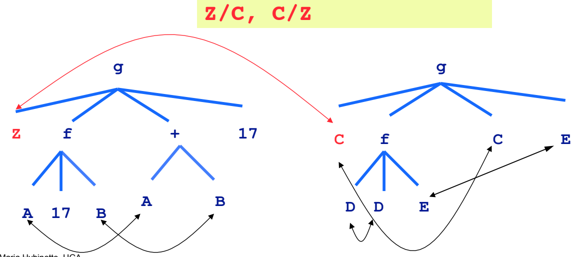
- First write in the co-referring variables.



Unify?

- Recursive descent: We go top-down, left-to-right
  - » but the order does not matter as long as it is systematic and complete.

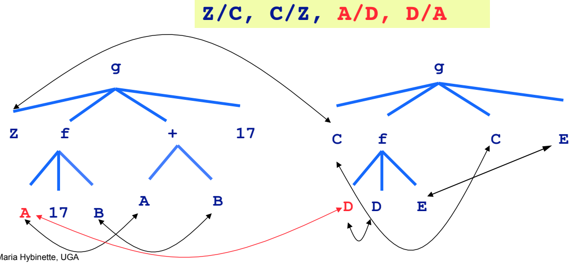
`Z/C, C/Z`



Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

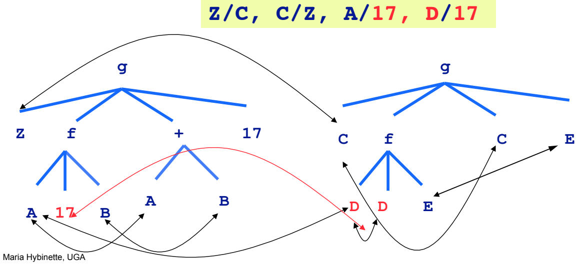
`Z/C, C/Z, A/D, D/A`



Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

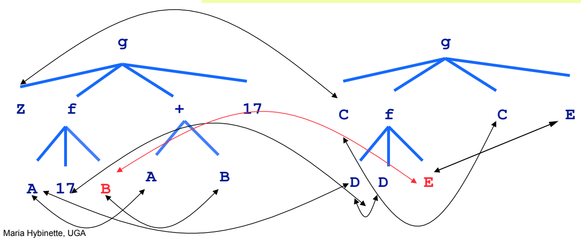
`Z/C, C/Z, A/17, D/17`



Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

Z/C, C/Z, A/17, D/17, B/E, E/B



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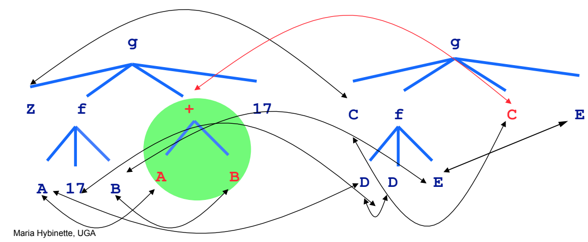
Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

Z/C, C/Z, A/17, D/17, B/E, E/B



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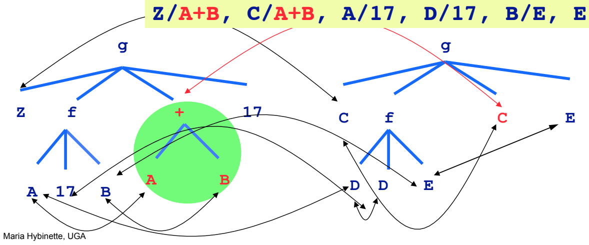
Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

Z/C, C/Z, A/17, D/17, B/E, E/B

Z/A+B, C/A+B, A/17, D/17, B/E, E/B



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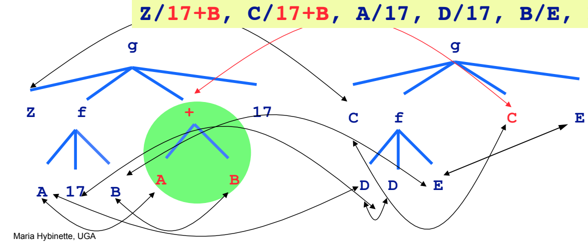

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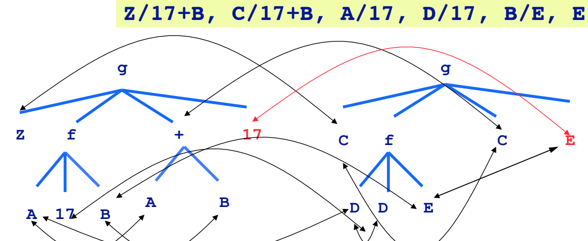

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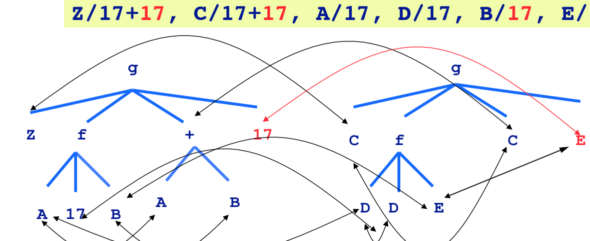
Unify?

- recursive descent We go top-down, left-to-right, but the order does not matter as long as it is systematic and complete.

Z/17+17, C/17+17, A/17, D/17, B/17, E/17



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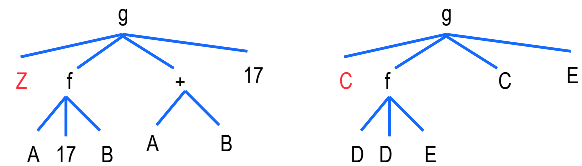
Can also use "substitution method"

Exercise – Alternative Method

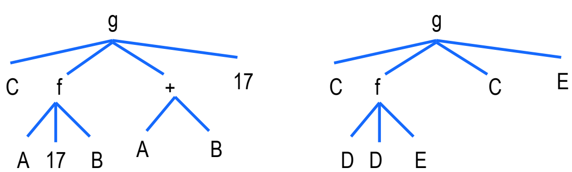
Z/C



Make 1st tree look like 2nd

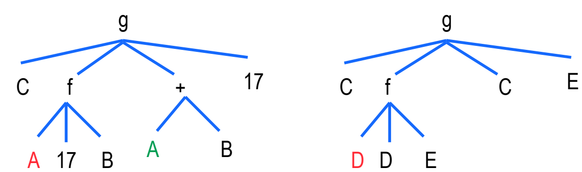
Exercise – Alternative Method

Z/C



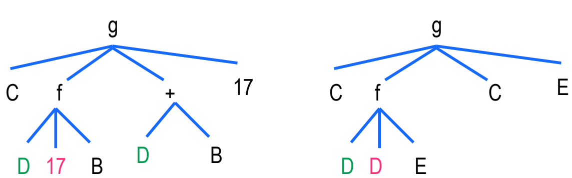
Exercise – Alternative Method

A/D, Z/C



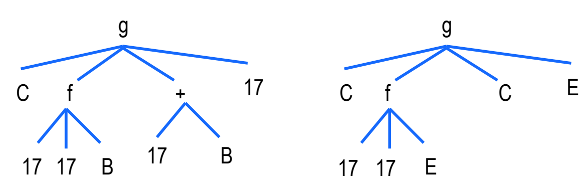
Exercise – Alternative Method

D/17, A/D, Z/C



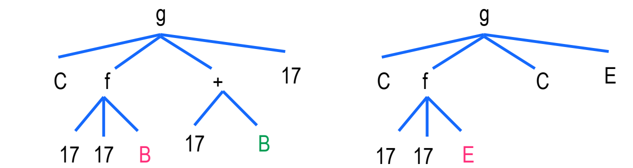
Exercise – Alternative Method

D/17, A/17, Z/C



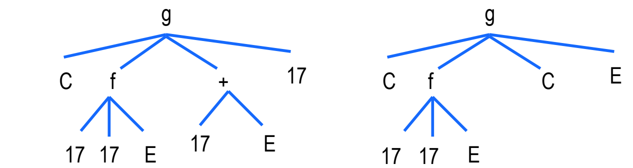
Exercise – Alternative Method

B/E, D/17, A/17, Z/C



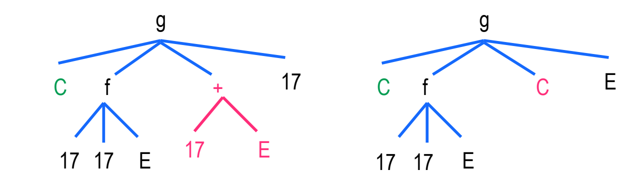
Exercise – Alternative Method

B/E, D/17, A/17, Z/C



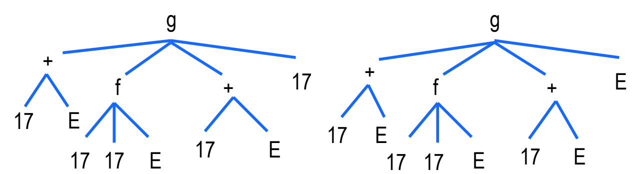
Exercise – Alternative Method

C/17+E, B/E, D/17, A/17, Z/C



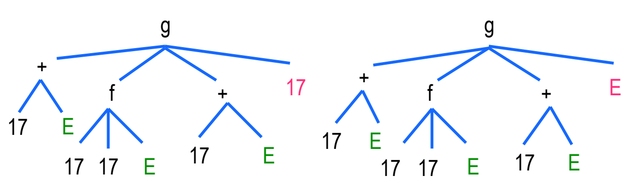
Exercise – Alternative Method

C/17+E, B/E, D/17, A/17, Z/17+E



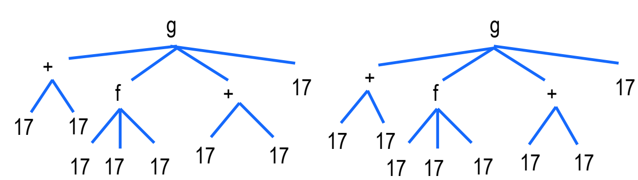
Exercise – Alternative Method

E/17, C/17+E, B/E, D/17, A/17, Z/C



Exercise – Alternative Method

E/17, C/17+17, B/17, D/17, A/17, Z/C



Operators

- Prolog has some predefined operators (and the ability to define new ones)
- An operator is just a predicate for which a special abbreviated syntax is supported
  - » Example: `+( 2, 3)` can also be written as `2 + 3`

The Predicate '='

- The goal `=(X,Y)` succeeds if and only if `X` and `Y` can be unified:

```
?- =(parent(maria,gunnar),parent(maria,X)).

X = gunnar

Yes
```

- Since `=` is an operator, it can be and usually is written like this:

```
?- parent(maria,gunnar)=parent(maria,X).

X = gunnar

Yes
```


The Predicate '='

- Note: The goal `=(X,Y)` succeeds if and only if `X` and `Y` can be unified. Consider =(5, +(3, 2))

```
?- (2+3) = 5.
No.
```


Arithmetic Operators

- Predicates `+`, `-`, `*` and `/` are operators too, with the usual precedence and associativity

```
?- X = +(1,*(2,3)).

X = 1+2*3

Yes
?- X = 1+2*3.

X = 1+2*3

Yes
```

Prolog lets you use operator notation, and prints it out that way, but the underlying term is still `+(1,*(2,3))`

Not Evaluated

```
?- +(X,Y) = 1+2*3.

X = 1
Y = 2*3

Yes
?- 7 = 1+2*3.

No
```

- The term is still `+(1,*(2,3))`
- It is not evaluated
- There is a way to make Prolog evaluate such terms…

Arithmetic

- *is* operator:
- `is(A, B / 10 + C)`
  - » `A is B / 10 + C`
- Unifies it's first argument with the arithmetic value of its second argument.
- Infix OK too: takes an arithmetic expression as right operand and variable as left operand
- Variables in the expression must all be instantiated.
  - » In above, B and C needs to have been instantiated.
- Variable on the left cannot be previously instantiated.
  - » In above A cannot be instantiated (what happens if A is not a variable?)

- Left hand side cannot be an expression since it is not evaluated -- it may be a value (and then unification is possible)

Unification impossible Example

- `Sum is Sum + Number`
- If `Sum` is not instantiated, the reference to its right is undefined and the clause fails
- If `Sum` is instantiated, the clause fails because the left operand cannot have a current instantiation when it is evaluated.

Arithmetic Evaluation `is/2`

```
?- X is 3 + 4.
X = 7

?- X = 3 + 4.
X = 3 + 4

?- 10 is 5 * 2. %
yes % b/c 10 is a “value”

?- 10 = 5 * 2.
no
```

```
?- is(X,1+2)
X=3
?- X is 1+2     % infix OK.
X=3
?- 1+2 is 4-1. % first argument
no             % already instantiated
?- X is Y.     % second argument Y
<error>                % must be
instantiated
?- Y is 1+2, X is Y.
X = 3          % Y instantiated
Y = 3          % before it is needed
```

- Unifies the first argument with the value of it’s second argument.
  - » In contrast to (=) unification predicate, which just unifies terms without evaluating them
- Note: left may not be a “variable” then it may unify with the value on the right.

Trace

- Built-in structure that displays instantiations at each step
- Tracing model of execution - four events:
  - » Call (beginning of attempt to satisfy goal)
  - » Exit (when a goal has been satisfied)
  - » Redo (when backtrack occurs)
  - » Fail (when goal fails)

`distance(chevy, Chevy_Distance). % Query`

Example Arithmetic

```
speed(ford,100).
speed(chevy,105).
speed(dodge,95).
speed(volvo,80).
time(ford,20).
time(chevy,21).
time(dodge,24).
time(volvo,24).
distance(X,Y) :-        speed(X,Speed),
                         time(X,Time),
                         Y is Speed * Time.
```


`distance(chevy, Chevy_Distance). % Query`

Example Arithmetic

```
speed(ford,100).
speed(chevy,105).
speed(dodge,95).
speed(volvo,80).
time(ford,20).
time(chevy,21).
time(dodge,24).
time(volvo,24).
distance(X,Y) :-        speed(X,Speed),
                         time(X,Time),
                         Y is Speed * Time.
```

```
trace.
distance(chevy, Chevy_Distance).
(1) 1 Call: distance(chevy, _0)?
(2) 2 Call: speed(chevy, _5)?
(2) 2 Exit: speed(chevy, 105)
(3) 2 Call: time(chevy, _6)?
(3) 2 Exit: time(chevy, 21)
(4) 2 Call: _0 is 105*21?
(2) 2 Exit: 2205 is 105 * 21
(1) 1 Exit: distance(chevy, 2205)

(2)
    Chevy_Distance = 2205
```


List Structures

- Other basic data structure (besides atomic propositions we have already seen): list
- List is a sequence of any number of elements
- List is a functor of arity 2,its first component is the head and the second is the tail.
- Elements can be atoms, atomic propositions, or other terms (including other lists)

Same as in Scheme

```
nil
(a, nil)
(a, .(b, nil)
(a, .(b, .(c, .(d, .(e. nil)))))
(a,b)  (note this is a pair, not a proper list)
(a, X) (this might be a list, or might not!)
(a, .(b, nil)), .(c, nil))
```


List Notation

- The lists is written using square brackets [].
- These are just abbreviations for the underlying term using the . Predicate
- List of length 0 is nil, denoted [].

```
?- X = .(1,.(2,.(3,[]))).

X = [1, 2, 3]

Yes
?- .(X,Y) = [1,2,3].

X = 1
Y = [2, 3]

Yes
```


List Notation and the Tail

List Notation	Term denoted
[1|X]	.(1,X)
[1,2|X]	.(1,.(2,X))
[1,2|[3,4]]	same as [1,2,3,4]

- [X | Y]
  - » X is bound to first element in list, the head.
  - » Y is bound to the remaining elements, called the tail.
- Useful in patterns: [1,2|X] unifies with any list that starts with 1,2 and binds X to the tail

```
?- [1,2|X] = [1,2,3,4,5].

X = [3, 4, 5]

Yes
```



```
[apple, prune, grape, kumquat]
[]          % (empty list)
[X | Y]     % (head X and tail Y)
```


The append Predicate

```
?- append([1,2],[3,4],Z).

Z = [1, 2, 3, 4]

Yes
```

- Predefined append(X,Y,Z) succeeds if and only if Z is the result of appending the list Y onto the end of the list X


```
?- append(X,[3,4],[1,2,3,4]).

X = [1, 2]

Yes
```

- append can be used with any pattern of instantiation (that is, with variables in any positions)

```
?- append(X,Y,[1,2,3]).

X = []
Y = [1, 2, 3] ;

X = [1]
Y = [2, 3] ;

X = [1, 2]
Y = [3] ;

X = [1, 2, 3]
Y = [] ;

No
```


Implementing append()

```
append([], List, List).
append([Head | List_1], List_2, [Head | List_3])
      :-  append (List_1, List_2, List_3).
```

- Suppose we want to join
  - » `[a, b, c]` with `[d, e]`.
  - » `[a, b, c]` has the recursive structure
    - `[a | [b, c] ]`.
  - » Then the rule says:
    - IF `[b,c]` appends with `[d, e]` to form `[b, c, d, e]`
    - THEN `[a|[b, c]]` appends with [d,e] to form `[a|[b, c, d, e]]`
      - » i.e. `[a, b, c]` `[a, b, c, d, e]`

Implementing append()

```
append([], List, List).
append([Head | List1], List2, [Head | List3])
      :-  append (List1, List2, List3).
```

- If you know that a particular `List1` will append with a `List2` to produce a `List3`,
  - » then you know how it will go for a case which is one step more complex.
    - a list which is one element longer (the `Head`). i.e. if you add a `Head` to `List1`, then the result of the append will be that `Head` on the front of `List3`.

Implementing append()

```
append([], List, List).
append([Head | List1], List2, [Head | List3])
      :-  append (List1, List2, List3).
```

```
?- append([a,b,c],[d],X).
append( [a, b, c], ....)
   IF append([b, c], ....)
      IF append([c], ....)
          IF append([], ....)

            append(...., [d])
          append(.... , [c,d])
      append(.... , [ b, c , d])
   append(.... , [ a, b , c ,d ])
```


Implementing append()

```
append([], List, List).
append([Head | List1], List2, [Head | List3])
      :-  append (List1, List2, List3).
```

```
?- append([a,b,c],[d],X).
append( [a, b, c], ....)
   IF append([b, c], ....)
      IF append([c], ....)
          IF append([], ....)

            append(...., [d])
          append(.... , [c,d])
      append(.... , [ b, c , d])
   append(.... , [ a, b , c ,d ])
```

```
append( [ a | [b,c]], [d], [a| NT1])
   IF append([b,c], [d], NT1)   X=[a| NT1]

append( [ b|[c]], [d], [b| NT2])
   IF append([c], [d], NT2)   NT1=[b| NT2]

append( [ c|[]], [d], [c| NT3])
   IF append([], [d], NT3)   NT2=[c|NT3]

append([],[d],[d])             NT3 = [d]

NT2 = [c | NT3] = [c|[d]} = [c,d]
NT1 = [b| NT2] = [b|[c,d]] = [b,c,d]
X   = [a|NT1] = [a|[b,c,d]] = [a,b,c,d]
```


Implementing append ()

```
append([], List, List).
append([Head | List_1], List_2, [Head | List_3])
      :-  append (List_1, List_2, List_3).
```

- Two first parameters the lists that are appended, the third parameters is the resulting list
- First proposition: when the empty list is appended to any other `list`
  - » the other list is the result.
- Second proposition:
  - » left hand side: first element of the new list (i.e. the result) is the same as the first element of the first given list (both are named `Head`).
  - » right hand side: the tail of the first given list (`List_1`) has the second given list (`List_2`) appended to form the tail of the resulting list.

```
append([], List, List).
append([Head | List_1], List_2, [Head | List_3])
      :-  append (List_1, List_2, List_3).
```

```
trace.
append([bob,jo], [jake, darcie], Family).

(1) 1 Call: append([bob, jo], [jake, darcie], _10)?
(2) 2 Call: append([jo], [jake, darcie], _18)?
(3) 3 Call: append([],[jake,darcie],_25)?
(3) 3 Exit: append([],[jake,darcie],[jake,darcie]))
(2) 2 Exit: append([jo],[jake,darcie],[jo,jake,darcie])
(1) 1 Exit: append([bob,jo],[jake,darcie,
   [bob,joe,jake,darcie])
Family = [bob, jo, jake, darcie]
```

Other Predefined List Predicates

Predicate	Description
`member(X,Y)`	Provable if the list **Y** contains the element **X**.
`select(X,Y,Z)`	Provable if the list **Y** contains the element **X**, and **Z** is the same as **Y** but with one instance of **X** removed.
`nth0(X,Y,Z)`	Provable if **X** is an integer, **Y** is a list, and **Z** is the **X**th element of **Y**, counting from 0.
`length(X,Y)`	Provable if **X** is a list of length **Y**.

- **All flexible, like `append`**
- **Queries can contain variables anywhere**

Using select

```
?- select(2,[1,2,3],Z).

Z = [1, 3] ;

No
?- select(2,Y,[1,3]).

Y = [2, 1, 3] ;

Y = [1, 2, 3] ;

Y = [1, 3, 2] ;

No
```



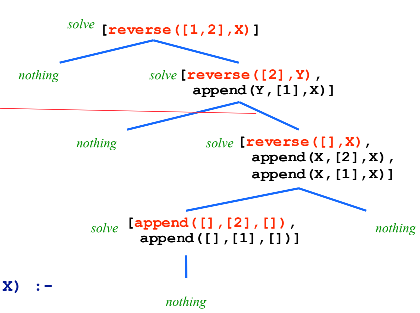
```
?- reverse([1,2,3,4],Y).

Y = [4, 3, 2, 1] ;

No
```

- **Predefined `reverse(X,Y)` unifies `Y` with the reverse of the list `X`**

- **Definition of reverse function:**

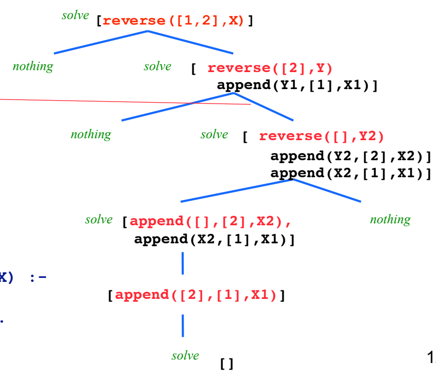
```
reverse([], []).
reverse([Head | Tail], X) :-
      reverse(Tail, Y),
      append(Result, [Head], X).
```


This step is wrong: we substituted **X** for **Y**, but there is already a different **X** elsewhere in the goal list.




```
reverse([],[]).
reverse([Head|Tail],X) :-
  reverse(Tail,Y),
  append(Y,[Head],X).
```

This step is wrong: we substituted **X** for **Y**, but there is already a different **X** elsewhere in the goal list.




```
reverse([],[]).
reverse([Head|Tail],X) :-
  reverse(Tail,Y),
  append(Y,[Head],X).
```


Deficiencies of Prolog

- Resolution order control
- The closed-world assumption
- The negation problem
- Intrinsic limitations

Advantages:

- Prolog programs based on logic, so likely to be more logically organized and written
- Processing is naturally parallel, so Prolog interpreters can take advantage of multi-processor machines
- Programs are concise, so development time is decreased – good for prototyping

SWI-Prolog

```
?- set_prolog_flag(history, 50).

Yes
27 ?- h.                    % shows history of commands
    2   eats(Person1,Food1).
    3   eats(Person1,Food),eats(Person2,Food).
    4   eats(corey,fish).
?- !!.                      % Repeats last query
```