Due: September 2, 2014

Directions: The below problems involve material from Ch 0–1 of the textbook. Submit your solutions during class or else to the instructor's office by noon on the given due date. Show all work performed. With the exception of graphs and figures, solutions should be typed (graphs and figures may be handwritten).

[Chapter 0]

1. For each part, give a relation that satisfies the condition. (15 points)

   (a) Reflexive and symmetric but not transitive.

   (b) Reflexive and transitive but not symmetric.

   (c) Symmetric and transitive but not reflexive.

   Assume the relation is defined on a nonempty set.

2. Consider the undirected graph $G = (V, E)$ where $V = \{1, 2, 3, 4\}$ and $E = (\{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\})$. Draw the graph. What is the degree of each node? Indicate a path from node 3 to 4 in your drawing of G. (8 points)

3. If A has a elements, B has b elements and C has c elements, how many elements are in $A \times B \times C$? Explain your answer. (7 points)

4. Show that every graph with 2 or more nodes contains two nodes that have equal degrees. (10 points)

[Chapter 1]

5. The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

6. Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 of the textbook[1] to give the state diagram of a DFA for the language given. Let $\Sigma = \{a, b\}$. (30 points)

   (a) $\{w | w$ has at least three a's and at least two b's$\}$.

   (b) $\{w | w$ has an odd number of a's and ends with a b $\}$.

   (c) $\{w | w$ starts with an a and has at most one b $\}$.

7. Prove that, if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA that recognizes the complement of B. Conclude that the class of regular languages is closed under complement.(10 points)

8. Let $D = \{w | w$ contains an even number of a's and an odd number of b's and does not contain the substring $ba\}$. Construct a DFA with five states that recognizes D. Assume $\Sigma = \{a, b\}$.(10 points)

[1] Page 46, Both 2nd ed and 3rd ed. The construction is mentioned in the proof that regular languages are closed under union.