Perception | Filtering: a worked example
Autonomous Mobile Robots

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Correlation in 2D

- Example:
  - Constant averaging filter

\[
F \circ I(x, y) = \sum_{j=-N}^{M} \sum_{i=-M}^{N} F(i, j) I(x+i, y+j)
\]

This example was generated with a 21x21 mask
Filtering | correlation in 2D

\[ F \circ I(x, y) = \sum_{j=-N}^{N} \sum_{i=-N}^{N} F(i, j) I(x+i, y+j) \]

- Example:
  Constant averaging filter

\[
F = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

- If \( size(F) = (2N + 1)^2 \) i.e. this is a square filter

- 2D Correlation \( \Rightarrow \) no. multiplications per pixel = ?
  no. additions per pixel = ?

This example was generated with a 21x21 mask
Filtering | correlation in 2D

\[ \text{No. multiplications per pixel} = (2N + 1)^2 \]

\[ \text{No. additions per pixel} = (2N + 1)^2 - 1 \]
**Filtering | correlation in 2D**

\[ F \circ I(x, y) = \sum_{j=-M}^{M} \sum_{i=-N}^{N} F(i, j) I(x+i, y+j) \]

- Example:
  Constant averaging filter
  
  \[
  F = \begin{bmatrix}
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9}
  \end{bmatrix} = \begin{bmatrix}
  \frac{1}{3} \\
  \frac{1}{3} \\
  \frac{1}{3}
  \end{bmatrix} \cdot \begin{bmatrix}
  \frac{1}{3} & \frac{1}{3} & \frac{1}{3}
  \end{bmatrix}
  
  \]

- If \( \text{size}(F) = (2N + 1)^2 \) i.e. this is a square filter

- 2D Correlation \( \Rightarrow \) no. multiplications per pixel = \((2N + 1)^2 \)
  no. additions per pixel = \((2N + 1)^2 - 1\)

- 2 \( \times \) 1D Correlation \( \Rightarrow \) no. multiplications per pixel = ?
  no. additions per pixel = ?
Filtering | correlation in 2D

⇒ No. multiplications per pixel so far = \(2N + 1\)

⇒ No. additions per pixel so far = \(2N\)
Filtering | efficient correlation in 2D

\[ \text{No. multiplications per pixel so far} = 2N + 1 \]
\[ \text{No. multiplications per pixel} = 2(2N + 1) \]

\[ \text{No. additions per pixel so far} = 2N \]
\[ \text{No. additions per pixel} = 4N \]
Filtering | efficient correlation in 2D

- Example:
  Constant averaging filter
  
  \[ F = \begin{bmatrix}
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
  \frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{bmatrix} \approx \text{"separable" filter}
\]

- If \( \text{size}(F) = (2N + 1)^2 \) i.e. this is a square filter
- 2D Correlation \( \Rightarrow \) no. multiplications per pixel \( = (2N + 1)^2 \)
  no. additions per pixel \( = (2N + 1)^2 - 1 \)
- \( 2 \times 1D \) Correlation \( \Rightarrow \) no. multiplications per pixel \( = 2(2N + 1) \)
  no. additions per pixel \( = 4N \)
- \( 2 \times 1D \) Correlation \( \Rightarrow \) no. multiplications per pixel \( = ? \)
  no. additions per pixel \( = ? \)
Filtering | more efficient correlation in 2D

**Perception | Filtering: a worked example**

- No. additions per pixel = $4N$
- No. multiplications per pixel = 1
Filtering | more efficient correlation in 2D

- Example:
  Constant averaging filter
  \[ F = \begin{bmatrix}
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
  \frac{1}{9} & \frac{1}{9} & \frac{1}{9}
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{bmatrix} \cdot \frac{1}{9} \begin{bmatrix} 1 \\
  1 \\
  1
\end{bmatrix} \]

  "separable" filter

- If \( \text{size}(F) = (2N+1)^2 \) i.e. this is a square filter
- 2D Correlation \( \Rightarrow \) no. multiplications per pixel = \((2N+1)^2\)
  no. additions per pixel = \((2N+1)^2 - 1\)
- 2 \times 1D Correlation \( \Rightarrow \) no. multiplications per pixel = \(2(2N+1)\)
  no. additions per pixel = \(4N\)
- 2 \times 1D Correlation \( \Rightarrow \) no. multiplications per pixel = \(1\)
  no. additions per pixel = \(4N\)