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Construction for proof of Thm 7.3.2 (omitting m' and commas in variable names).

Given PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, let $G = G(M) = (V, \Sigma, P, S)$ where

$V = \{S\} \cup \{\langle pAq \rangle : A \in \Gamma \cup \{\lambda\} \text{ and } p, q \in Q\}$

and P consists of the following rules:

1. $S \to \langle q_0 \lambda p \rangle$ for all $p \in F$;

2. for each transition $q \xrightarrow{x\,C/B} p$ in δ,

   $\langle qCr \rangle \longrightarrow x \langle pBr \rangle$ for all $r \in Q$

   and if $C = B = \lambda$

   $\langle qAr \rangle \longrightarrow x \langle pAr \rangle$ for all $A \in \Gamma$, $r \in Q$

   and (type 3 rules in text) if $C = \lambda$, $B \in \Gamma$

   $\langle qAr_2 \rangle \longrightarrow x \langle pBr_1 \rangle \langle r_1 A r_2 \rangle$

   for all $A \in \Gamma$, r_1 and $r_2 \in Q$;

4. $\langle p \lambda p \rangle \longrightarrow \lambda$ for all $p \in Q$.

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Example 7.3.1 (using 0 for q_0, 1 for q_1):

1. $S \to \langle 0 \lambda 1 \rangle$

2. from $\begin{tikzcd} q_0 \arrow[loop right, "a\ \lambda/A"] \end{tikzcd}$

$\langle 0 \lambda 0 \rangle \to a \langle 0 A 0 \rangle$,

and $\langle 0 \lambda 1 \rangle \to a \langle 0 A 1 \rangle$,

<u>and</u> (type 3 rules)

$\langle 0 A 0 \rangle \to a \langle 0 A 0 \rangle \langle 0 A 0 \rangle$, $(r_1 = r_2 = q_0)$

$\langle 0 A 1 \rangle \to a \langle 0 A 0 \rangle \langle 0 A 1 \rangle$, $(r_1 = q_0, r_2 = q_1)$

$\langle 0 A 1 \rangle \to a \langle 0 A 1 \rangle \langle 1 A 1 \rangle$, $(r_1 = r_2 = q_1)$

$\langle 0 A 0 \rangle \to a \langle 0 A 1 \rangle \langle 1 A 0 \rangle$; $(r_1 = q_1, r_2 = q_0)$

from $q_0 \xrightarrow{c\ \lambda/\lambda} q_1$

$\langle 0 \lambda 0 \rangle \to c \langle 1 \lambda 0 \rangle$,

and $\langle 0 \lambda 1 \rangle \to c \langle 1 \lambda 1 \rangle$,

<u>and</u> $\langle 0 A 0 \rangle \to c \langle 1 A 0 \rangle$,

$\langle 0 A 1 \rangle \to c \langle 1 A 1 \rangle$;

from $\begin{tikzcd} q_1 \arrow[loop right, "b\ A/\lambda"] \end{tikzcd}$

$\langle 1 A 0 \rangle \to b \langle 1 \lambda 0 \rangle$,

and $\langle 1 A 1 \rangle \to b \langle 1 \lambda 1 \rangle$;

4. $\langle 0 \lambda 0 \rangle \to \lambda$,

$\langle 1 \lambda 1 \rangle \to \lambda$.

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Example 7.3.1 continued. Simplify G, first by removing useless symbols.

step	terminable
0	$\langle 0\lambda 0\rangle, \langle 1\lambda 1\rangle$
1	$\langle 1A1\rangle, \langle 0\lambda 1\rangle$
2	$\langle 0A1\rangle$, S
3	— (done)

$\therefore$ $\langle 1\lambda 0\rangle, \langle 0A0\rangle$, and $\langle 1A0\rangle$ are not terminable.

After removing $\langle 1\lambda 0\rangle, \langle 0A0\rangle$ and $\langle 1A0\rangle$:

$$S \to \langle 0\lambda 1\rangle$$
$$\langle 0\lambda 1\rangle \to a\langle 0A1\rangle \mid c\langle 1\lambda 1\rangle$$
$$\langle 0A1\rangle \to a\langle 0A1\rangle\langle 1A1\rangle \mid c\langle 1A1\rangle$$
$$\langle 1A1\rangle \to b\langle 1\lambda 1\rangle$$
$$\langle 0\lambda 0\rangle \to \lambda$$
$$\langle 1\lambda 1\rangle \to \lambda$$

step	reachable
0	S
1	$\langle 0\lambda 1\rangle$
2	$\langle 0A1\rangle, \langle 1\lambda 1\rangle$
3	$\langle 1A1\rangle$
4	— (done)

$\therefore$ $\langle 0\lambda 0\rangle$ is not reachable; so we omit it.

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Example 7.3.1 concluded.

Note that $\langle 1\lambda 1\rangle$ can only generate λ, so can replace $\langle 1\lambda 1\rangle$ by λ.

Then note that $\langle 1A1\rangle$ can only generate b, so can replace $\langle 1A1\rangle$ by b.

Resulting grammar:

$$S \to \langle 0\lambda 1\rangle$$
$$\langle 0\lambda 1\rangle \to a\langle 0A1\rangle \mid c$$
$$\langle 0A1\rangle \to a\langle 0A1\rangle b \mid cb$$

So $\langle 0A1\rangle$ generates $\{a^n c b^{n+1} : n \geq 0\}$

S and $\langle 0\lambda 1\rangle$ generate $\{a^{n+1} c b^{n+1} : n \geq 0\} \cup \{c\}$

$= \{a^m c b^m : m \geq 0\}$; this is $L(G)$ and $L(M)$.