DETECTION OF HAIRLINE MANDIBULAR FRACTURE USING MAX-FLOW MIN-CUT AND KOLMOGOROV-SMIRNOV DISTANCE

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ABSTRACT

This paper addresses the clinically challenging problem of hairline mandibular fracture detection from a sequence of Computed Tomography (CT) images. A hairline fracture of critical clinical importance, can be easily missed due to the absence of sharp surface and contour discontinuities and the presence of intensity inhomogeneity in the CT image, if not scrutinized carefully. In this work, the 2D CT image slices displaying a mandible with hairline fractures are first identified within an input CT image sequence of a fractured craniofacial skeleton. This is achieved via an intensity-based image retrieval scheme using Kolmogorov-Smirnov distance as the measure of similarity and an unbroken mandible as the reference image. Since a hairline fracture is essentially a discontinuity in the bone contour, we model it as a minimum cut in an appropriately weighted flow network. The existing graph cut-based segmentation schemes are enhanced with a novel construction of the flow network, guided by the geometry of the human mandible. The Edmonds-Karp refinement of the classical Ford-Fulkerson algorithm is employed next to obtain a minimum cut, which represents the hairline fracture in the already identified CT image slices. Experimental results demonstrate the effectiveness of the proposed method.

***Index Terms*—** Hairline mandibular fracture, Max-flow min-cut, Kolmogorov-Smirnov distance.

1. INTRODUCTION

Mandibular fractures occur frequently due to gunshot wounds, motor vehicle accidents, sports-related and battlefield-related injuries [1]. The mandible is usually less protected and hence more vulnerable than other parts of the human anatomy, even with full-body armor. The term *hairline fracture* refers to situations where the broken bone fragments are not visibly out of alignment and exhibit very little relative displacement. Detection of hairline mandibular fractures is particularly challenging due to the absence of sharp surface or contour discontinuities and the presence of intensity inhomogeneities in a CT scan. Thus, there is a significant probability that such fractures could escape manual scrutiny even if performed by a trained radiologist.

Computer-aided fracture detection has gained popularity over the past decade. Some recent works in this area include use of texture for the detection of hip fractures by Yap *et al.* [2], detection of fractures in femur bones using a combination of classifiers by Lum *et al.* [3], use of active contour modeling coupled with shape constraints for the detection of fractures in the arm by Jia and Jiang [4], femur fracture detection via a divide-and-conquer approach in kernel-space using a Support Vector Machine (SVM) by He *et. al.* [5] and the use of the Hough transform and gradient analysis for the detection of midshaft long-bone fractures by Donnelley *et. al.* [6]. Although mandibular fractures are encountered frequently, there is a relative paucity of work in computer-aided detection of such fractures. Detection of well-displaced mandibular fractures, wherein the broken fragments have suffered noticeable relative displacement (in sharp contrast to the present scenario), using a Bayesian inference approach has been reported in previous work by Chowdhury *et al.* [7]. A Markov Random Field (MRF)-based approach for detection of mandibular hairline fractures has been previously presented by Chowdhury *et al.* [8]. However, the MRF-based scheme described in [8] is computationally intensive. Furthermore, the MRF-based scheme is based on an underlying assumption that the bilateral symmetry of a mandible is preserved in the case of a hairline fracture, which may not necessarily hold in some cases.

In this paper, the 2D CT slices containing a fractured mandible are first identified within the entire CT image sequence of a fractured craniofacial skeleton using the Kolmogorov-Smirnov (KS) distance measure. Within each identified 2D CT slice, a hairline fracture is modeled as a *cut* in the flow of image intensities along the bone contours between two anatomical landmark points on the human mandible, called condyles. The geometry of the mandibular contours is exploited to build the flow network. In graph theory, this translates to finding a minimum cut in the flow graph between the *source* and the *sink* vertices. A fracture is detected by determining a minimum cut using the Edmonds-Karp enhanced version of the classical max-flow min-cut algorithm of Ford and Fulkerson [9]. Unlike the MRF-based scheme [8], the proposed method is computationally fast and does not rely on the assumption of preservation of bilateral symmetry for mandibles with hairline fractures.

The rest of the paper is organized in the following manner: in Section 2, we discuss how the relevant CT slices containing a fractured mandible are retrieved using the KS distance. In Section 3, we describe the process of fracture detection using the max-flow min-cut algorithm. In Section 4, we present the experimental results. Finally, the paper is concluded in Section 5 with an outline of future research directions.

2. RETRIEVAL OF RELEVANT CT SLICES USING KOLMOGOROV-SMIRNOV DISTANCE

The input to our problem is a sequence of CT images showing a fractured human craniofacial skeleton. Typically, a fractured mandible appears only in a few CT slices within the entire sequence. An intensity-based image retrieval technique is employed using the Kolmogorov-Smirnov (KS) distance as a measure of (dis)similarity to identify the fracture-containing 2D CT slices. Various distance measures have been used for capturing the (dis)similarity in the content of two images in content-based image retrieval (CBIR) systems [10]. A 2D CT slice (appearing in the middle of the CT sequence) of an unbroken craniofacial skeleton containing the intact mandible is selected as the reference CT slice. Each CT slice appearing in an input CT sequence of a broken craniofacial skeleton containing mandibular hairline fractures is treated as a candidate CT slice. The histograms of the pixel intensities of the reference CT slice and each candidate CT slice are first computed. The respective histograms are treated as probability distribution functions (pdfs) from which the cumulative distribution functions (cdfs) are then obtained. The KS distance measure is used to quantify the (dis)similarity between the cdf of the reference CT slice and the cdf of each candidate CT slice. Let $X(x)$ and $Y(x)$ respectively denote the pdfs of the reference CT slice and a candidate CT slice. Let $X^c(x)$ and $Y^c(x)$ respectively denote the cdfs corresponding to $X(x)$ and $Y(x)$. The KS distance between these two cdfs is given by:

$$KS(X^c(x), Y^c(x)) = \sup_x |X^c(x) - Y^c(x)| \quad (1)$$

The candidate CT slices which yield a smaller KS distance from the reference slice than a pre-determined threshold are deemed to contain the fractured mandible. An important advantage of using the KS distance measure is its non-parametric nature, thus no particular form of distribution is necessary for its computation [11]. Furthermore, the KS distance is also found to be robust to outliers.

3. FRACTURE DETECTION USING MAX-FLOW MIN-CUT

In this section, we first discuss the construction of the flow network for a 2D CT slice with suitable choices of vertices, edges and capacity functions to be used as edge weights. We will then justify our claim that the fracture detection problem can be mathematically modeled as being equivalent to the identification of a minimum cut in the constructed flow network.

3.1. Construction of the flow network

In their formulation of a flow network, Boykov and Jolly [12] choose all the pixels in an image as the vertices and establish edge connections amongst the 8 pixel neighbors. For the present problem, a flow network $G = (V, E)$ is constructed differently. It is noticed that a typical hairline fracture appears along the bone contours. So, we choose the set of boundary pixels P on the two mandibular contours (i.e. the inner and the outer contour) as the vertices of the proposed flow network. The two condyles, which are anatomical landmark points at the two terminals of the jaw, serve as natural choices for the source vertex s and the sink vertex t. Thus, we can write: $V = P \cup \{s, t\}$.

We now explain the process of identifying the mandibular contour points and the two condyles in a 2D CT slice for which some domain knowledge about the appearance of human mandibles in the CT scans is used. Specifically, (i) an intensity threshold for a mandibular

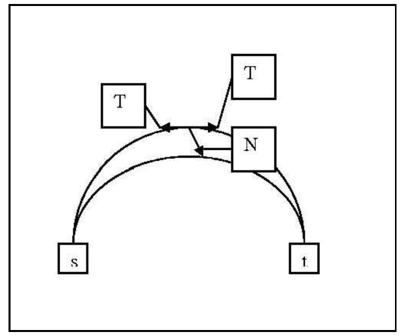

Fig. 1. A 2D flow network with a source s, a sink t and representative tangential (T) and normal (N) edges.

pixel and (ii) a bounding box for a mandible are assumed. Within the bounding box, we employ a boundary following algorithm [13] on the pixels which exceed the intensity threshold. Both the threshold value and the bounding box are kept constant for all the experimental datasets. The two condyles are two bottom-most points in the two bilateral halves of the image. Thus, we locate the condyles by identifying the two points on the mandibular contours with a maximum value of the y-coordinate (with respect to the image coordinate system) in the two halves of a 2D CT slice.

Since the boundary pixels for a mandible lie essentially on an arc, we construct *tangential* (T) and *normal* (N) edges in the proposed flow network representation. For each boundary pixel on a specific mandibular contour, we create edge links with the immediately forward and backward neighboring pixels. These constitute the T edges. On the other hand, the N edges are established between any two normal (or near-normal) boundary points across the two contours q and r. In addition the first boundary pixel of both the contours (q_f, r_f) are attached to the source (s) and the last boundary pixel of both the contours (q_l, r_l) are attached to the sink (t). Therefore,

$$E = T \cup N \cup \{(q_f, s), (r_f, s), (q_l, t), (r_l, t)\}$$

The rationale of having tangential and normal neighbors is guided by the geometry of the mandible as well as by the fracture pattern. A typical hairline fracture appears along and also across the two mandibular contours (for example see Figures (3) - (4)). The tangential edges are appropriate for capturing a fracture along the contour and the normal edges are suitable for identifying a fracture across the contour. So a fracture is appropriately modeled by using both types of edges. Let j and k be any two consecutive points along a mandibular contour with coordinates (x_j, y_j) and (x_k, y_k) respectively. Then the equation of the line that is normal to the contour at point j is given by:

$$(x_k - x_j)x + (y_k - y_j)y + (x_j(x_j - x_k) + y_j(y_j - y_k)) = 0 \quad (2)$$

In order to be a normal neighbor to the point j, a point on the other contour should ideally satisfy equation (2). However, it is not always possible to find the exact normal neighbor (primarily due to sampling error). So, we compute the distance d_{jm} of a set of competing candidate points m having coordinates (m_x, m_y) and choose the one which yields the minimum value of d_{jm}. From basic coordinate geometry, we obtain:

$$d_{jm} = \frac{Am_x + Bm_y + C}{\sqrt{A^2 + B^2}} \quad (3)$$

where $A = (x_k - x_j)$, $B = (y_k - y_j)$ and $C = (x_j(x_j - x_k) + y_j(y_j - y_k))$. We choose a simple capacity function, with intensity and distance as its two parameters, as an edge weight between a pair of pixels. Let I_p and I_q be the intensities of two pixels j and k and let d_{jk} be the Euclidean distance between them. Then the capacity function c_{jk} is given by:

$$c_{jk} = \frac{I_j I_k}{d_{jk}} \quad (4)$$

Since a typical fracture is characterized by the loss of bone material, the fracture site has a lower intensity value than the surrounding bone in a CT scan. Additionally, the distance between two boundary pixels would be relatively higher if it encompasses a fracture site. This justifies the choice of the capacity function, (given by equation (4)) since both tangential and normal edges exhibit lower capacity values at fracture sites. Very high capacity values are assigned to the edges which connect q_f and r_f to the source s and those which connect the sink t to q_l and r_l. By construction, all edges in the proposed flow network have a capacity value > 0. A schematic diagram of a 2D flow network with source and sink vertices, and tangential and normal edges is shown in Figure(1).

3.2. Correctness of the Modeling

We provide an intuitive justification for the correctness of the proposed modeling of fracture detection as a graph cut. We follow the framework of Boykov and Jolly [12]. From the discussion in the previous section, it is evident that every cut C in the flow network G satisfies following two properties:

1. C groups the vertices of G into two disjoint sets.
2. One set will contain the source vertex s and the other set will contain the sink vertex t.

The following theorem is essential to justify the correctness of the proposed network flow model. We state this theorem without proof. The interested reader can refer to [9] for a detailed proof.

Theorem. For any graph the maximum flow value from source vertex s to sink vertex t is equal to the minimal cut capacity of all cuts separating s and t. This is known as the Max-flow Min-Cut theorem.

Claim: A minimum cut C^* correctly identifies a fracture in the proposed 2D flow network G.

Justification: The justification is based on the above theorem and the construction of the flow network with the capacity function given by equation (4). We seek to determine the maximum flow between s and t. Using the above theorem, we obtain the minimum cut C^*. The minimum cut will consist of the cut edges in our flow network G. Basically, the cut edges are edges with comparatively low capacity values. From equation (4), it is evident that the low capacity edges are edges with relatively lower pixel intensity values and relatively higher distance values. These are exactly the characteristics of an edge in the vicinity of a fracture site. Note that since the broken fragments in a hairline fracture are not visibly out of alignment, a non-zero flow exists between them albeit with a lower value. The drop in the flow value is due to the presence of the fracture, which is a flow bottleneck in the constructed flow network. Thus, identification of a minimum cut C^* corresponds to detection of a fracture in the 2D flow network G.

The computation of the augmenting path is done using a *breadth-first search*, as outlined in the *Edmonds-Karp* enhancement [14] of the classical *Ford-Fulkerson* algorithm which has a worst-case time complexity $O(|V||E|^2)$, where $|V|$ denotes the cardinality of the vertex set V and $|E|$ denotes the cardinality of the edge set E in the flow network G.

4. EXPERIMENTAL RESULTS

The threshold value for the KS distance is experimentally chosen to be 0.25. Thus, only those CT slices which yield a value of KS distance lower than that of the threshold, are retrieved. Some results of KS distance-based image retrieval are shown in Figure (2). In this figure, the two fracture-containing CT slices out of a total of four are the ones which are similar in appearance to the reference CT slice and are hence retrieved. Note that the max-flow min-cut algorithm is applied exclusively to those CT slices which are retrieved. We now show both qualitatively and quantitatively the performance of the fracture detection procedure using the 2D max-flow min-cut algorithm. This algorithm was applied to 43 slices, retrieved from 10 data sets. Two different mandibles with the fracture sites identified are shown in Figures (3) and (4). In each of these figures, the centers of the crosses mark the source vertex (in the left half of the image) and the sink vertex (in the right half of the image) and the fractures are indicated by dark squares. Upon execution of the max-flow min-cut algorithm, we obtain the edges in the cut set. Each such edge joins a vertex on the *source*-side with another vertex on the *sink*-side of the flow network. For proper visualization, each such vertex is represented by a black square. It is evident from the figures that the fractures are identified accurately in both cases. The ground-truth is obtained via manual detection by trained radiologists. We achieve a sensitivity of 79% and a specificity of 59% after comparing our results with the ground-truth. The above values of sensitivity and specificity can be ascribed to i) use of a simple capacity function during the min-cut and ii) use of only the intensity feature in the image retrieval procedure. The execution time for the above 2D max-flow min-cut algorithm is only few seconds on a 1.73 GHz $Intel^{©}$ $Pentium^{©} M$ Processor.

5. CONCLUSIONS AND FUTURE WORK

Mandibular hairline fractures are difficult to detect (in the CT scans) due to the absence of sharp surface and contour discontinuities. In this paper, we model a hairline fracture as a minimum cut in an appropriately weighted flow network. The flow network is constructed based on the geometry of the human mandible and some prior knowledge of the fracture pattern. A simple capacity function is used to compute the edge weights. The Edmonds-Karp enhanced Ford-Fulkerson algorithm is employed on the 2D flow network to obtain a minimum cut. We have so far achieved a sensitivity of 79% and a specificity of 59%. To the best of our knowledge, this is the first detailed study of computer-aided detection of mandibular hairline fractures. The present work serves as an important tool for the radiologists and the craniofacial surgeons. From the computer vision perspective, the work represents an elegant blend of nonparametric statistics, geometry and graph theory.

In future, we plan to make the capacity function more robust by incorporating anatomical knowledge. For example, we can use the information that tissue swelling and specific low intensity regions called emphysema [15] typically appear in the vicinity of a mandibular fracture. The fracture detection results can be also improved by incorporating shape features in the KS distance-based retrieval process. Another direction of future research would be to detect a hairline fracture in 3D by applying the max-flow min-cut algorithm on all the retrieved CT slices (in a given dataset) taken together.

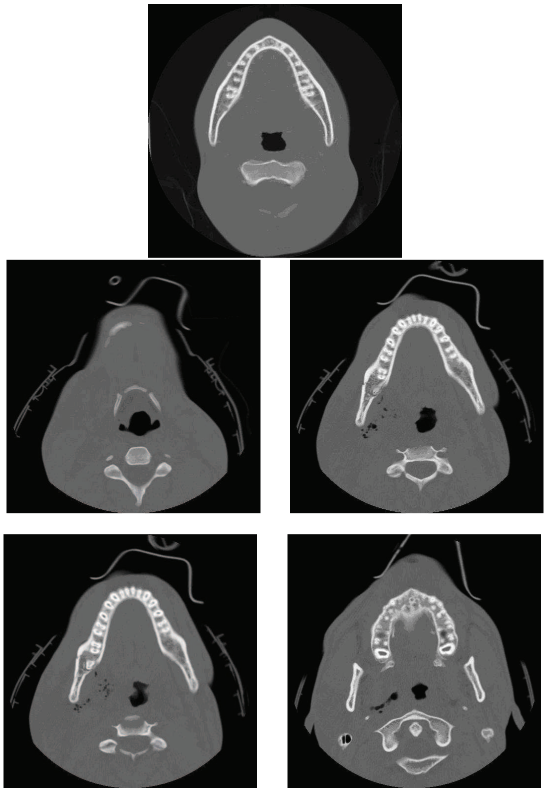

Fig. 2. Results from KS distance-based image retrieval for four slices from one CT image sequence. The unbroken reference mandible is shown in the first row. The right CT slice in the second row and the left slice in the third row are the ones retrieved.

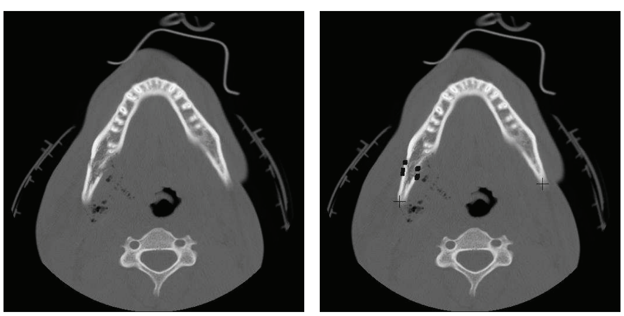

Fig. 3. 2D slice of a fractured mandible in the left. Fracture detection (black squares) with source and sink identification (black crosses) in the right.

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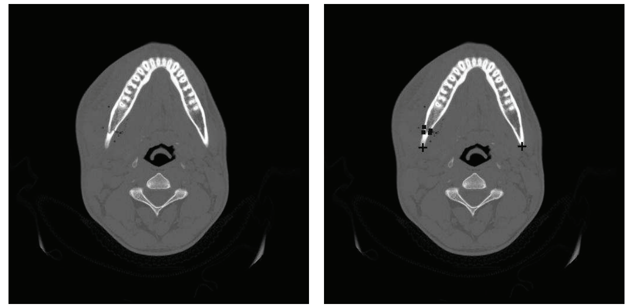

Fig. 4. 2D slice of a fractured mandible in the left. Fracture detection (black squares) with source and sink identification (black crosses) in the right.

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