

5.4 No. For example, consider $A = \{0^n 1^n \mid n = 0, 1, 2, 3, \dots\}$ and let f be the function that maps $0^n 1^n$ to 1^n and maps all other string to -1 . Then f is a mapping reduction from A to 1^* , which is regular even though A is not regular.

5.5 Show that A_{TM} is not mapping reducible to E_{TM} . I will be using the \neg symbol to indicate complement instead of drawing a line over the language. We know that A_{TM} is Turing recognizable, but not co-Turing recognizable. The TM below recognizes $\neg E_{TM}$, so E_{TM} is co-Turing recognizable.

$M =$ "On input $\langle M \rangle$, where M is a TM

1. For each $i = 1, 2, 3, \dots$

1. Run M on all strings of length i for i steps

2. If any string is accepted, **accept**"

This Turing machine will accept any Turing machine whose language is non-empty.

Now assume A_{TM} is mapping reducible to E_{TM} . Then $\neg A_{TM}$ is mapping reducible to $\neg E_{TM}$. But, $\neg E_{TM}$ is Turing recognizable and $\neg A_{TM}$ is not, which contradicts Theorem 5.22. This is a contradiction. Therefore, A_{TM} is not mapping reducible to E_{TM} .

5.7 $A \leq_m \neg A$ implies $\neg A \leq_m A$. By Theorem 5.16, we can conclude that $\neg A$ is Turing recognizable since we know A is Turing recognizable. By Theorem 4.16, we can conclude A is decidable since it is both TR and co-TR.

5.3 Let A be any Turing recognizable language and let M be a Turing machine such that $L(M) = A$. Let f be the function that maps any string w to the string $\langle M.w \rangle$. Then w is in A if and only if $f(w)$ is in A_{TM} – i.e., f is a mapping reduction from A to A_{TM} .

6.3 Since $A \leq_T B$, there is a Turing machine M_1 that calls an oracle for B and decides A . Similarly, since $B \leq_T C$, there is a Turing machine M_2 that calls an oracle for C and decides B . Now, consider the Turing machine M that does exactly what M_1 does except instead of calling the oracle for B , it calls M_2 . Since M_2 decides B , it will give the same answer as the oracle did, so M will decide A . Also, M_2 uses an oracle for C , so M also uses an oracle for C to decide A . Therefore, $A \leq_T C$.