Use the pumping lemma to prove $A = \{ww^R | w \in \{a,b\}^*\}$ is not regular.

Proof: Assume $A$ is regular. Then $A$ has an associated pumping length $p$ such that any string $s$ in $A$ with $|s| \geq p$ can be written as $s = xyz$ such that

1. $|xy| \leq p$
2. $|y| > 0$
3. $xy^iz \in A$ for every $i = 0, 1, 2, \ldots$

Consider the string $s = a^p bba^p$. Since $s = ww^R$, where $w = a^p b$, $s$ is in $A$. Therefore the pumping lemma holds and $s = xyz$ for some $x$, $y$, and $z$ with the above properties. Since $xy$ is a prefix of $s$ with at most $p$ symbols, it must be the case that $xy = a^k$ for some $k \leq p$. Also, since $y$ is a substring of $xy$, $y$ must be $a^j$ for some $j = 1, 2, \ldots, k$ ($j$ cannot be 0 by property 2 above).

Now consider the string $xz$. This string is the result of “pumping” $y$ 0 times. $xz = a^{p-j} bba^p$. Since $j > 0$, $xz$ is not in $A$. Thus, the pumping lemma fails, which contradicts our assumption that $A$ is a regular language.

1.54 a. $F$ cannot be regular because if a string in $F$ starts with exactly 1 $a$, it must be followed by $b^n c^n$ and we know that there is no DFA that accepts $a^n b^n$.

b. $F$ acts like a regular language with pumping length 2. Given any string $s$ of length at least 2, you can always pump the string as follows:

- If $s = aab^n c^m$ where $n \neq m$, pump the string $aa$ (i.e., let $x = \varepsilon$ and $y = aa$). Then $xy^i z = a^{2i} b^n c^m$ which is in $A$ since $a^{2i}$ cannot equal $a$.
- If $s$ is anything else, let let $x = \varepsilon$ and $y = the first symbol in s.$
  - If $s = ab^n a^n$, then $xy^i z = a^ib^n c^n$, which is still in $A$ for any value of $i$
  - If $s = a^{2+x} b^n c^m$, where is some positive integer, then $xy^i z = a^{2+x+i-1} b^n c^m$, which is still in $A$ since $2+x+i-1 > 1$
- If $s$ starts with a $b$ or a $c$ then $xy^i z$ simply pumps the first symbol, which would result in a string still in $A$

c. The above argument does not contradict the pumping lemma because the pumping lemma does not claim that if a language can be pumped then it is a regular language. I simply claims that if a language is regular then it can be pumped.
2.1 \text{E} \rightarrow \text{E+T} \rightarrow \text{E+T+T} \rightarrow \text{T+T+T} \rightarrow \text{F+F+F} \rightarrow \text{a+a+a}
2.1 d $E \rightarrow T \rightarrow F \rightarrow (E) \rightarrow (T) \rightarrow (F) \rightarrow ((E)) \rightarrow ((T)) \rightarrow ((F)) \rightarrow ((a))$

2.4 c The idea: Have 2 variables $S$ and $T$. $S$ is used to add a single symbol to an even length string. When a derivation contains a $T$, it has an odd number of terminals. Therefore, you can stop (replace $T$ with $\varepsilon$) or add a symbol and go back to the $S$ variable.

$$S \rightarrow 0T \mid 1T$$
$$T \rightarrow 0T \mid 1S \mid \varepsilon$$

2.4 e The idea: Add symbols in pairs on either side of your variable. Therefore, your palindrome is growing from the center.

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$

2.6 b The complement of $a^n b^n$ is the union of the following languages:

- $a^n b^n$, where $n > m$. A CFG that accepts this language is
  - $S_1 \rightarrow aS_1 \mid aT_1$
  - $T_1 \rightarrow aT_1 b \mid \varepsilon$
- $a^n b^n$, where $n < m$. A CFG that accepts this language is
  - $S_2 \rightarrow S_2 b \mid T_2 b$
  - $T_2 \rightarrow aT_2 b \mid \varepsilon$
- strings that start with a $b$. A CFG that accepts this language is
Strings that have a “b” followed by an “a”. A CFG that accepts this language is

- $S_3 \rightarrow bT_3$
- $T_3 \rightarrow aT_3 \mid bT_3 \mid \varepsilon$

Therefore, a CFG that accepts the complement of $a^*b^n$ is

$S \rightarrow S_1 \mid S_2 \mid S_3 \mid S_4$

where $S_1$, $S_2$, $S_3$ and $S_4$ are defined above.

2.6 d  We want to create a palindrome that might have any of the following things added to it (1) some string ending with a # preceding it, (2) some string beginning with a # following it, or (3) some string bracketed by # in the middle of it. The variable A below creates any string in \{0,1,#\}*. P creates our palindrome, R adds a string ending with a # to the beginning of the string, T adds a string beginning with a # to the end, and M inserts a string bracketed by # symbols into the middle of the palindrome.

$$
S \rightarrow RPT \\
P \rightarrow 0P0 \mid 1P1 \mid 1 \mid 0 \mid \varepsilon \mid M \\
R \rightarrow A# \mid \varepsilon \\
T \rightarrow #A \mid \varepsilon \\
M \rightarrow #A# \mid \varepsilon \\
A \rightarrow 0A \mid 1A \mid #A \mid \varepsilon $$