Homework 6
Solution
CSCI 2670
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3.7 The problem is with step 2. The Turing machine has to evaluate the polynomial at *every* possible combination of integer values for $x_1, x_2, \ldots, x_k$. Since there are an infinite number of possibilities, the machine will never get past this step and accept if the polynomial has some integer root.

3.15 b) Assume we have two decidable languages A and B. We want to show that A°B is also decidable. Let M_1 and M_2 be two deciders such that $L(M_1) = A$ and $L(M_2) = B$. Consider the following Turing machine:

C = "On input w

Let $l = length(w)$
For each $i = 0, 1, 2, \ldots, l$
    Let w_l = the leftmost i symbols in w
    Let w_r = the rightmost $l-i$ symbols in w
    Run M_1 on input w_l
    If it accepts
        Run M_2 on input w_r
        If it accepts **accept**
Next i
reject"

Claim: C decides A°B. This is clear since if w is in A°B, there is some i for which w_l is in A and w_r is in B. When the loop reaches that i, C will accept. If w is not in A°B, then no iteration of the loop will cause C to accept so C will reject as soon as it exits the loop. Note that since M_1 and M_2 are both deciders, C *will* eventually reach an accept or reject state.

3.15 c) Assume A is a decidable language and M decides A. Let C be the decider that decides A°A. Consider the following Turing machine:

S = "On input w

Let $l = length(w)$
If $l = 0$ **accept**
Submit w to M (the TM that decides A)
    If M accept w **accept**
For each $i = 1, 2, \ldots, l$
    Let w_l = the leftmost i symbols in w
    Let w_r = the rightmost $l-i$ symbols in w
    Run M on input w_l
    If it accepts

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            Run S on input w_r (recursive call)
            If it accepts accept
Next i
reject”
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Claim: S decides A*. This is clear since if w is in A*, then either $w = \varepsilon$ or there is some i for which w_l is in A and w_r is in A*. When the loop reaches that i, S will accept. If w is not in A*, then no iteration of the loop will cause S to accept so S will reject as soon as it exits the loop. Note that since M is a decider, no calls to M prevent S from reaching a halting state. Also, every recursive call to S has a shorter input string, so the recursion will not cause an infinite loop. Therefore, the loop will eventually complete and S will reject any string not in A*.

3.16 d) Assume A and B are Turing-recognizable languages. Let M_1 and M_2 be Turing machines that recognize A and B, respectively. Consider the following Turing machine:

Int = “On input w

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1   Submit w to M1
2         If M1 accepts w
3         Submit w to M2
4               If M2 accepts w accept
5   reject”
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Claim: Int recognizes A∩B. This is clear since if w is in A and in B, Int will be able to arrive at line 4 and accept w. If w is either not in A or not in B, and both M_1 and M_2 halt on input w, then Int will reject w. However, if either M_1 or M_2 do not halt on input w, then Int will not halt on w. This does not matter, though, since in this case w will not be in A∩B and Int is only required to be a recognizer – i.e., Int does not have to halt on strings that are not in A∩B.