CSCI 2670 Intro. to Theory of Comp.
Quiz 8
11/3/05

Name: _____

In this quiz, you will prove the following language is undecidable

$FINITE_{TM}$ = {<M> | M is a TM and L(M) is finite}.

The quiz has 4 questions. If you skip a question, you can answer the next question as though you had answered the previous one. For example, in (2) you are asked to create a Turing machine P and in (3) you need to use P. You can use P in (3) even if you couldn't successfully create P in (2).

Assume $FINITE_{TM}$ is decidable and that F decides it.

(1) (1 point) Pick any finite language, call it FL.

FL = **{0,00,000}**

(2) (3 points) Create a Turing machine P satisfying the following property:

L(P) = Σ* If M accepts w
L(P) = FL Otherwise

You do not need to argue that this property is satisfied. I have written the first line of P for you.

P = "On input x:
1. If x ∈ FL, *accept*
2. Run M on input w
3. If M accepts w, *accept*"

(3) (3 points) Using F (the decider for $FINITE_{TM}$) and P (your answer to (2)), create a decider S for A_{TM}. You don't need to argue that S is a decider or that $L(S)=A_{TM}$.

S = "On input <M,w>, where M is a TM

1. Create TM P from question (2)
2. Submit <P> to TM F
3. If F accepts <P>, *reject*
4. If F rejects <P>, *accept*"

(4) (3 point) Based on the decider presented in (3), what can you conclude? Why does your conclusion follow?

$FINITE_{TM}$ is not decidable. This follows because (3) demonstrates that if $FINITE_{TM}$ is decidable, then A_{TM} is also decidable, but we know that A_{TM} is undecidable.