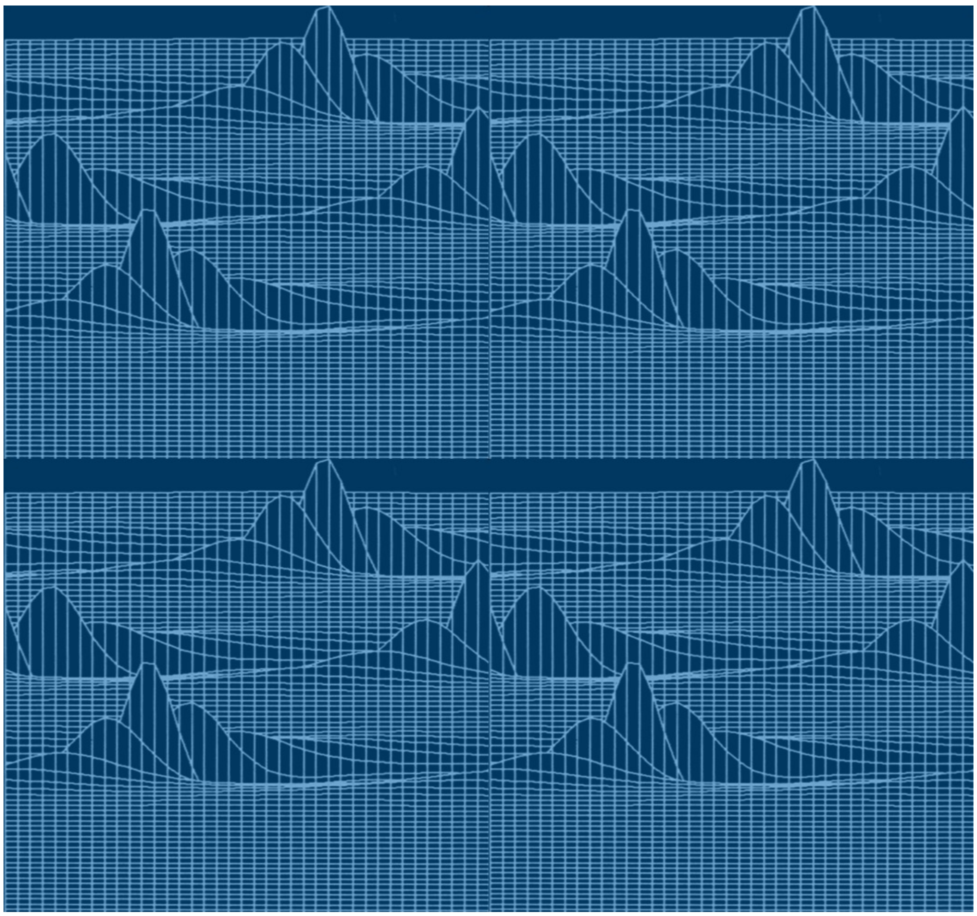
The Thirteenth International Conference on

NONLINEAR EVOLUTION EQUATIONS AND WAVE PHENOMENA: COMPUTATION AND THEORY

The Classic Center, Athens, Georgia - USA

April 14–16, 2025

BOOK OF ABSTRACTS



http://waves2025.uga.edu/

Edited by Gino Biondini and Thiab Taha

2025.4.13

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School of Computing, University of Georgia

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Chuntao Yin (Shijiazhuang Tiedao U.)
Runzhang Xu (Harbin Engineering U.)
Yi Zhu (China)

Keynote talks

Peter Engels (Washington State University):
“Ultracold Quantum Gases: A Prime Testbed for Studying Nonlinear Dynamics”

Mark Hoefer (University of Colorado Boulder):
“Recent Developments in Dispersive Hydrodynamics”

Vera Hur (University of Illinois at Urbana-Champaign):
“Advancing Stability through Rigorous Computation”

The up-to-date conference program is available at http://waves2025.uga.edu/program.shtml

PROGRAM AT A GLANCE

Monday, April 14, 2025

	Athena Ballroom E	*Ballroom A*	*Ballroom B*	*Ballroom C*	*Ballroom D*
8.10–8.30am	*Welcome*				
8.30–9.30am	Keynote lecture I: Mark Hoefer				
9.30–9.55am	*Coffee break*				
10.00–10.50am	S20 - I/III	S9	Papers	S21 - I/IV	S27 - I/II
10.55am–12.10pm	S2 - I/II	S11 - I/III	S15	S26	S22 - I/II
12.10–1.40pm	*Lunch (attendees on their own)*				
1.40–3.20pm	S2 - II/II	S16	S17	S10	S13 - I/II
3.20–3.50pm	*Coffee break*				
3.50–5.55pm	S19	S11 - II/III	S20 - II/III	S21 - II/IV	S12 - I/III

Tuesday, April 15, 2025

	Athena Ballroom E	*Ballroom A*	*Ballroom B*	*Ballroom C*	*Ballroom D*
8:00–9:00am	Keynote lecture 2: Vera Hur				
9:10–10:00am	S8 - I/V	S5 - I/II	S7	S23 - I/III	S21 - III/IV
10:00–10:30am	*Coffee break*				
10:30–12:10pm	S13 - II/II	S5 - II/II	S14	S23 - II/III	S1 - I/II
12:10–1:40pm	*Lunch (attendees on their own)*				
1:40–3:20pm	S8 - II/V	S6 - I/II	S24 - I/II	S11 - III/III	S18 - I/II
3:20–3:50pm	*Coffee break*				
3:50–5:55pm	S8 - III/V	S6 - II/II	S25 - I/II	S27 - II/II	S12 - II/III
7:00–9:00pm	*Conference banquet (including student papers award)*				

Wednesday, April 16, 2025

	Athena Ballroom E	*Ballroom A*	*Ballroom B*	*Ballroom C*	*Ballroom D*
8:00–9:00am	Keynote lecture 3: Peter Engels				
9:10–10:00am	S8 - IV/V	S4 - I/II	S18 - II/II	S24 - II/II	S23 - III/III
10:00–10:30am	*Coffee break*				
10:30–12:10pm	S8 - V/V	S4 - II/II	S25 - II/II	S1 - II/II	S21 - IV/IV
12:10–1:40pm	*Lunch (attendees on their own)*				
1:40–3:20pm	S12 - III/III	Papers	S20 - III/III	Papers	S22 - II/II

PROGRAM

Sunday, April 13, 2025

5:00–7:00 **REGISTRATION AND RECEPTION**, Athena Ballroom A

Monday, April 14, 2025

7:30–10:00 **REGISTRATION**, Athena Ballroom A
8:10–8:30 **WELCOME**
Thiab Taha, Program Chair and Conference Coordinator
8:30–9:30 **KEYNOTE LECTURE I**, Athena Ballroom E
Mark Hoefer: *Recent developments in dispersive hydrodynamics*
Chair: Thiab Taha

9:30– 9:55 COFFEE BREAK

10:00–10:50 **SESSION 20**, Athena Ballroom E: Investigation of nonlinear dispersive wave systems - I/III
Chairs: Min Chen, Olivier Goubet and Bingyu Zhang
10:00–10:25 Jerry Bona: *A model for bore propagation with dynamic boundary conditions*
10:25–10:50 Shenghao Li, Xin Yang and Bingyu Zhang:
Conditional and unconditional local well-posedness problem for some dispersive systems on the half line

10:00–10:50 **SESSION 9**, Ballroom A: Recent trends in acoustics and related wave phenomena: computational and analytical methodologies, Chair: James Lambers
10:00–10:25 James Lambers: Krylov Subspace Spectral Methods for Problems in Acoustics
10:25–10:50 Chandler Shimp and James Lambers: Numerical simulation of finite amplitude acoustic waves featuring gradient catastrophe

10:00–10:50 **SESSION 21**, Ballroom C: Advances in integrable systems and inverse scattering - I/IV
Chairs: Barbara Prinari and Gino Biondini
10:00–10:25 Kaitlynn Lilly and Tom Trogdon:
A numerical Riemann-Hilbert approach to the computation of transform pairs
10:25–10:50 Francesco Demontis: *Three-way focusing Zakharov-Shabat systems with zero diagonal entry*

10:00–10:50 **SESSION 27**, Ballroom D: Theoretical and numerical advances in fluids and nonlinear optics - I/II
Chairs: Denis Silantyev, Sergey Dyachenko and Alexandr Chernyavsky
10:00–10:25 Justin Cole: *Topological insulators in magneto-optical systems*
10:25–10:50 Rafail Abramov: *Turbulence via intermolecular potential*

10:00–10:50 **PAPERS**, Ballroom B: Nonlinear partial differential equations and dynamical behavior of solutions, Chair: Runzhang Xu
10:00–10:25 S. Sangeetha, K. Mathiyalagan, Sangmoon Lee and Ju H. Park: *Stabilization and SMC design for coupled ODE-transport PDE Systems*
10:25–10:50 Shashwat Sharan, Patrick Sprenger, Boaz Ilan and Mark Hoefer: *Vacuum dam-break problem for the nonlinear Schrodinger equation in a harmonic potential*

10:55–12:10 **SESSION 2**, Athena Ballroom E: Evolution equations and integrable systems - I/II
Chairs: Alex Himonas, Curtis Holliman and Fangchi Yan
10:55–11:20 Alex Himonas and Fangchi Yan: *Initial-boundary value problems for NLS equations*
11:20–11:45 Alex Himonas and Fangchi Yan: *A higher order quadratic NLS equation on the half-line*
11:45–12:10 Andreas Chatziafratis, Jerry Bona, Hongqiu Chen and Spyridon Kamvissis: *The linear BBM-equation on the quarter-plane, revisited: A novel rigorous approach and unexpected phenomena*

10:55–12:10 **SESSION 11**, Ballroom A: Water waves - I/III, Chairs: John Carter and Bernard Deconinck
10:55–11:20 Katie Oliveras: *Multiscale formulation of water waves: derivation, modeling, and stability analysis*
11:20–11:45 Eleanor Byrnes: *High Amplitude Stokes Waves in a Finite Depth Fluid*

10:55–12:10 **SESSION 15**, Ballroom B: Recent advances in stability of nonlinear waves
Chairs: Robert Marangell, Jared Bronski and Graham Cox
10:55–11:20 Mitchell Curran and Robert Marangell: *The fourth-order nonlinear Schrodinger equation and the Maslov index*
11:20–11:45 Dave Smith: *Linearized KdV equation on the line with a metric graph defect*
11:45–12:10 Hewan Shemtaga, Wenxian Shen and Selim Sukhtaiev: *Well-posedness, stability, and bifurcation for Keller-Segel models on compact graphs*

10:55–12:10 **SESSION 26**, Ballroom C: Waves in complex mathematical models of optics and superconductivity
Chairs: Alejandro Aceves and Jean Guy Caputo
10:55–11:20 Jacek Gatlik, Tomasz Dobrowolski, and Panayotis G. Kevrekidis: *Kink dynamics in modified sine-Gordon model*
11:20–11:45 Jean-Guy Caputo: *Mathematical models of flux trapping by superconductors*
11:45–12:10 Austin Marsteller: *Nonlinear dynamics in non-local Ablowitz lattice models*

10:55–12:10 **SESSION 22**, Ballroom D, Nonlinear waves in lattices - I/II
Chairs: Christopher Chong and Timothy Faver
10:55–11:20 Christopher Chong: *Dispersive shock waves in granular chains*
11:20–11:45 Andrew Hofstrand: *Bridging opposing asymptotic descriptions of discrete breathers on nonlinear lattices*
11:45–12:10 Dmitry Pelinovsky: *Existence of generalized breathers and transition fronts in time-periodic nonlinear lattices*

12:10–1:40 LUNCH AND REGISTRATION

1:40–3:20 **SESSION 2**, Athena Ballroom E: Evolution equations and integrable systems - II/II
Chairs: Alex Himonas, Curtis Holliman and Fangchi Yan
1:40–2:05 Timur Akhunov and Ángel David Martínez:
Illposedness of KdV-type equations using the modified energy method
2:05–2.30 Curtis Holliman and Francesca Cozzi: *Current issues regarding the modeling of glioblastomas*
2:30–2:55 Guher Camliyurt: *Global well-posedness and scattering results for nonlinear wave equations*
2:55–3:20 John Holmes: *Nonlinear diffusion equations on the half line*

1:40–3:20 **SESSION 16**, Ballroom A: Recent developments on free surface flows
Chairs: Robin Ming Chen, Samuel Walsh and Miles Wheeler
1:40–2:05 Huy Nguyen, Ryan Creedon and Walter Strauss: *Proof of the transverse instabilities of Stokes wave*
2:05–2.30 Jörg Weber: *On bounds for the Froude number for solitary water waves*
2:30–2:55 Chen-Chih Lai and Michael Weinstein: *Thermal effects on the deformation of a gas bubble in an incompressible liquid*
2:55–3:20 Robin Ming Chen, Kristoffer Varholm, Samuel Walsh and Miles Wheeler: *Vortex-carrying solitary gravity waves of large amplitude*

1:40–3:20 **SESSION 17**, Ballroom B: Fractional calculus and its application, Chair: Yang Liu
1:40–2:05 Changpin Li: *H3N3-2σ-based finite difference method for the fractional diffusion-wave equations*
2:05–2.30 Yang Liu: SCQ for fractional calculus
2:30–2:55 Yue Cao: *Global Mittag-Leffler stability of the delayed fractional-coupled reaction-diffusion system on networks without strong connectedness*
2:55–3:20 Guoyu Zhang: *A high-order discrete energy decay and maximum-principle preserving scheme for time fractional Allen–Cahn equation*

1:40–3:20 **SESSION 10**, Ballroom C: Recent advances in nonlinear differential equations and applications

Chairs: Andrei Ludu, Harihar Khanal and Adrian Carstea

1:40–2:05 Andrei Ludu: *Nonlinear Liouville problem for the distribution of ice in the Arctic*
2:05–2.30 Oleksandr Bobrovnikov and Alexei Rybkin: *Inverse problems for tsunami waves*
2:30–2:55 Huaxia Liu and Yongbing Luo: *G-Neutral functional equations with impulsive: existence, attracting and stability*
2:55–3:20 Dhruba Adhikari: *Inclusions involving perturbed positively homogeneous maximal monotone operators*

1:40–3:20 **SESSION 13**, Ballroom D: Nonlinear partial differential equations and dynamical behavior of solutions - I/II, Chairs: Wei Lian and Runzhang Xu

1:40–2:05 Runzhang Xu: *Global quantitative stability of wave equations with strong and weak dampings*
2:05–2.30 Wei Lian and Erik Wahlén: *Transverse instability of line periodic waves to the KP-I equation*
2:30–2:55 Xingchang Wang: *Equivalence of weak formulations of stratified steady water waves*
2:55–3:20 Zhuang Han: *Qualitative behavior for one-dimensional sixth-order Boussinesq equation with logarithmic nonlinearity*

3:20–3:45 COFFEE BREAK

3:50–5:55 **SESSION 19**, Athena Ballroom E: Stability for nonlinear waves with singularity
Chairs: Geng Chen and Yannan Shen

3:50–4:15 Siming He, Stability of Close-to-Couette Shear Flows in a Finite Channel
4:15–4:40 Xin Liu, Edriss Titi and Claude Bardos, Fast-slow dynamics of the quasi-geostrophic approximation
4:40–5:05 Geng Chen, L2 theory for compressible fluid
5:05–5:30 Yannan Shen, Stability of dispersive shock for the KdV Burgers' equation
5:30–5:55 Quyuan Lin, Stable Singularity Formation of the Inviscid Primitive Equations

3:50–5:55 **SESSION 11**, Ballroom A: Water waves - II/III, Chairs: John Carter, Bernard Deconinck

3:50–4:15 Andre Nachbin: *Faraday waves and the spontaneous synchronization of oscillators*
4:15–4:40 John Carter, Diane Henderson and Panos Panayotaros: *The spatial Whitham equation*
4:40–5:05 Anastassiya Semenova: *Two-crested Stokes waves*
5:05–5:30 Joanna van Liew, John Carter and Diane Henderson: *Modeling broad-frequency-band water-wave experiments*
5:30–5:55 Henrik Kalisch: *Collapsing breakers*

3:50–5:55 **SESSION 20**, Ballroom B: Investigation of nonlinear dispersive wave systems - II/III
Chairs: Min Chen, Olivier Goubet , and Bingyu Zhang

3:50–4:15 Xin Yang, Shenghao Li and Bing-Yu Zhang: *Effect of lower order terms on the well-posedness of Majda-Biello systems*
4:15–4:40 Deqin Zhou: *Global well-posedness, ill-posedness and long time behavior of solutions of the surface electromigration equation*
4:40–5:05 Mathieu Colin and Tatsuya Watanabe: *On a Schrodinger-Poisson system with doping profile*
5:05–5:30 Shu-Ming Sun: *On solitary-wave solutions of generalized abcd-Boussinesq system with a Hamiltonian structure*
5:30–5:55 Angel Duran: *Existence and stability of solitary wave solutions of Boussinesq-Full Dispersion systems for internal waves*

3:50–5:55 **SESSION 21**, Ballroom C: Advances in integrable systems and inverse scattering - II/IV
Chairs: Barbara Prinari and Gino Biondini

3:50–4:15 Deniz Bilman: *Infinite-order solutions of the AKNS hierarchy and their asymptotic behavior under the mKdV flow*
4:15–4:40 Peter Miller: *The explicit solution of the Benjamin-Ono equation with general rational (including non-soliton) initial data*
4:40–5:05 Aikaterini Gkogkou: *Soliton gas for the focusing nonlinear Schrödinger equation with box-like initial conditions*

5:05–5:30 *Chris Mayo: Well-posedness of a higher-order nonlinear Schrodinger equation on a finite interval*
5:30–5:55 *Joanne Dong and Peter Miller: New results in the Riemann-Hilbert problem for the semiclassical defocusing Schrodinger equation*

3:50–5:55 **SESSION 12**, Ballroom D: Singular asymptotics for integrable nonlinear waves and related topics, I/III
Chairs: Deniz Bilman, Robert Buckingham and Peter Miller
3:50–4:15 Manuela Girotti, Tamara Grava, Ken McLaughlin and Joseph Najnudel: *Random solitons, soliton gasses, and all that*
4:15–4:40 Cade Ballew, Thomas Trogdon and Deniz Bilman: *Computing KdV soliton gas solutions*
4:40–5:05 Samir Donmazov, Jiaqi Liu and Peter Perry: *Long-time asymptotics for the KP I equation with small initial data*
5:05–5:30 Roozbeh Gharakhloo and Tomas Lasic Latimer: *Graph enumeration via random matrix models, a report on some new results*
5:30–5:55 Alexander Moll: *Borodin-Olshanski z-measures from the renormalization of quantum Benjamin-Ono periodic traveling waves*

Tuesday, April 15, 2025

7:30–9:30 **REGISTRATION**, Athena Ballroom A
8:00–9:00 **KEYNOTE LECTURE II**, Athena Ballroom E
Vera Hur: *Advancing stability through rigorous computation*
Chair: Gino Biondini

9:10–10:00 **SESSION 8**, Athena Ballroom E: Geometric methods in spectral theory of traveling waves - I/V
Chairs: Mitch Curran, Yuri Latushkin and Selim Sukhtaiev
9:10–9:35 Keith Promislow and Brian Wetton: *Faceting and folding in sharp interface gradient flows*
9:35–10:00 Stephane Lafortune: *On the stability of smooth solutions to peakon equations*

9:10–10:00 **SESSION 5**, Ballroom A: Peakons, kinkons and other weak solutions for nonlinear wave equations - I/II
Chairs: Stephen Anco, Zhijun Qiao and Vesselin Vatchev
9:10–9:35 Ranis Ibragimov: *Invariant solutions of nonlinear mathematical modeling of natural phenomena*
9:35–10:00 Zhijun Qiao: *Peaked solitons and beyond*

9:10–10:00 **SESSION 7**, Ballroom B: Recent and alternate methods for the numerical solution of partial differential equations, Chairs: Muhammad Usman and Chaudry Masood Khalique
9:10–9:35 Jacob Shapiro and Amanda Criner: *Spectral representations in thermography*
9:35–10:00 Arunasalam Rahunanthan, Abdullah Al Mamun, Alsadig Ali, Abdullah Al-Mamun and Felipe Pereira: *A novel multiscale sampling algorithm for subsurface characterization*

9:10–10:00 **SESSION 23**, Ballroom C: Painlevé equations, integrable systems, and related topics - I/III
Chairs: Anton Dzhamay, Pieter Roffelsen and Alexander Stokes
9:10–9:35 Dmitry Korotkin: *Tau-functions and monodromy symplectomorphisms*
9:35–10:00 Tomas Lasic Latimer: *Positive solutions to discrete Painleve equations and orthogonal polynomials*

9:10–10:00 **SESSION 21**, Ballroom D: Advances in integrable systems and inverse scattering - III/IV
Chairs: Barbara Prinari and Gino Biondini
9:10–9:35 Nicholas Ossi, Evans Boadi, Efstathios Charalampidis, Panayotis Kevrekidis and Barbara Prinari: *Kuznetsov-Ma breather solutions of the defocusing Ablowitz-Ladik equation with large background amplitude*
9:35–10:00 Baofeng Feng and Changyan Shi: *A Sasa-Satsuma-mKdV equation and its various soliton solutions*

10:00–10:25 COFFEE BREAK

10:30–12:00 **SESSION 13**, Athena Ballroom E: Nonlinear partial differential equations and dynamical behavior of solutions - II/II, Chairs: Wei Lian and Runzhang Xu

10:30–10:55 Yitian Wang: *Well-posedness for p(x)-Laplacian parabolic equations with multiple regime on an annulus*

10:55–11:20 Bastian Hilder, Erik Wahlen and Giang To: G*lobal bifurcation of three-dimensional gravity-capillary waves on Beltrami flows*

11:20–11:45 Stefano Böhmer and Dag Nilsson: *Solitary axisymmetric capillary water waves*

11:45–12:10 Yue Pang: *Global existence and finite time blowup for an anisotropic parabolic equation in weighted variable Sobolev spaces*

10:30–12:00 **SESSION 5**, Ballroom A: Peakons, kinkons and other weak solutions for nonlinear wave equations - II/II Chairs: Stephen Anco, Zhijun Qiao and Vesselin Vatchev

10:30–10:55 Thomas Wolf and Stephen Anco: *Polynomial evolution equations over the octonions with Lax pairs*

10:55–11:20 Vesselin Vatchev: *On power series method for nonlinear fluid dynamics equations*

11:20–11:45 Lung-Hui Chen: *Uniqueness of inverse spectral problem of non-local Sturm-Liouville operators on star graph*

11:45–12:10 Stephen Anco and Zhijun Qiao: *Peakons on a kink background*

10:30–12:00 **SESSION 14**, Ballroom B: Recent advances in algorithmic development for nonlinear PDEs Chairs: Jia Zhao, Xuelong Gu and Qi Wang

10:30–10:55 Xuejian Li, John Singler and Xiaoming He: *Incremental data compression for PDE-constrained optimization*

10:55–11:20 Xinfeng Liu: *Exponential time differencing method for a reaction- diffusion system with free boundary*

11:20–11:45 Xuelong Gu: *Parallel and energy-preserving schemes based on the partitioned averaged vector field method*

11:45–12:10 Jia Zhao: G*eneral numerical framework for structure-preserving reduced order models of thermodynamically consistent reversible-irreversible PDEs*

10:30–12:00 **SESSION 23**, Ballroom C: Painlevé Equations, Integrable Systems, and Related Topics - II/III Chairs: Anton Dzhamay, Pieter Roffelsen and Alexander Stokes

10:30–10:55 Robert Buckingham: *Asymptotics of Umemura rational Painleve-V functions*

10:55–11:20 Haru Negami: *Integral transformations of KZ-type equations and the construction of unitary representations of braid groups*

11:20–11:45 Kouichi Takemura: *Symmetry of q-Painleve equations and Lax pairs*

11:45–12:10 Andrii Liashyk: *Integrable spinning fluid*

10:30–12:00 **SESSION 1** , Ballroom D: Nonlinear Waves and Application–Part I/II Chairs: Jerry Bona, Hongqiu Chen

10:30–10:55 Hongqiu Chen: *The long wavelength limit of periodic solutions of water wave models*

10:55–11:20 Jose Vega-Guzman: *Minimum-uncertainty squeezed states for a time-dependent Schrodinger equation*

11:20–11:45 Junsik Bae, Sang-Hyuck Moon and Kwan Woo: *Emergence of peaked singularities in the Euler-Poisson system*

11:45–12:10 Yoshihiro Ueda: *Stability of stationary waves for viscoelastic fluids in half-space*

12:10–1:40 LUNCH AND REGISTRATION

1:40–3:20 **SESSION 8**, Athena Ballroom E: Geometric methods in spectral theory of traveling waves - II/V Chairs: Mitch Curran, Yuri Latushkin and Selim Sukhtaiev

1:40–2:05 Ming Chen, Yang Lan, Yue Liu and Zhong Wang: *Asymptotic stability of smooth solitons for the Camassa-Holm equation*

2:05–2:30 Alin Pogan and Yuri Latushkin: *representations of strongly continuous cosine operator families in terms of the eigenvalues and the resonances of the generator*

2:30–2:55 Ryan Creedon, Walter Strauss and Huy Nguyen: *Transverse instability of Stokes waves*

2:55–3:20 John Zweck, Erika Gallo and Yuri Latushkin: *Spectral stability via the Fredholm determinant of a trace class Birman-Schwinger operator*

1:40–3:20	**SESSION 6**, Ballroom A: Recent developments in nonlinear waves: From rogue waves and blow-ups to shocks, vortices and beyond - I/II
	Chairs: Efstathios Charalampidis, Ricardo Carretero-Gonzalez and Panayotis Kevrekidis
1:40–2:05	Roy Goodman and Atul Anurag: *The phase space of the three-vortex problem*
2:05–2:30	Efstathios Charalampidis: *Spectral analysis of blow-up in nonlinear dispersive equations: Theory and computations*
2:30–2:55	Rahul Kashyap, Satya N. Majumdar and Surajit Sen: *Rogue energy fluctuations as extreme events may be the hallmark of non-integrable nonlinear systems*
2:55–3:20	Gino Biondini: *Two-dimensional dispersive shock waves in the Kadomtsev-Petviashvili equation*

1:40–3:20	**SESSION 24**, Ballroom B: Recent Advances in photonic systems - I/II
	Chairs: Justin Cole and Ziad Musslimani
1:40–2:05	Savvas Sardelis, Ziad Musslimani, Andrea Blanco-Redondo, Shuva Roy and Mrinmoy Roy: *Pure-quartic solitons with PT-symmetric nonlinearity*
2:05–2:30	Alexander Cerjan: *Local topological classification of open and nonlinear photonic systems*
2:30–2:55	Junshan Lin: *Edge modes in topological photonic crystals*
2:55–3:20	Michael Nameika and Justin Cole: *Modeling 1D topological insulators in the presence of noisy data*

1:40–3:20	**SESSION 11**, Ballroom C: Water waves - III/III
1:40–2:05	Bernard Deconinck: *The Benjamin-Feir instability in KdV-like equations with general dispersion and monomial nonlinearity*
2:05–2:30	Xinyu Zhao, David Nicholls and Jon Wilkening: *Spatially quasi-periodic water waves*
2:30–2:55	Levent Batakci, Bernard Deconinck and David Nicholls: *The instabilities of 2D periodic traveling water waves*
2:55–3:20	Sergey Dyachenko: *The Stokes waves on ideal fluid: modulational instability and wave breaking*

1:40–3:20	**SESSION 18**, Ballroom D: Existence and stability of traveling waves - I/II
	Chairs: Anna Ghazaryan, Stephane Lafortune and Vahagn Manukian
1:40–2:05	Yuri Latushkin and Alin Pogan: *Uniform bounds of families of analytic semigroups and Lyapunov linear stability of planar fronts*
2:05–2:30	Ross Parker: *Multi-modal solitary wave solutions to a fourth-order nonlinear Schrodinger equation*
2:30–2:55	Qiliang Wu: *Existence of asymmetric grain boundaries*

3:20–3:45 COFFEE BREAK

3:50–5:55	**SESSION 8**, Athena Ballroom E: Geometric methods in spectral theory of traveling waves - III/V
	Chairs: Mitch Curran, Yuri Latushkin and Selim Sukhtaiev
3:50–4:15	Jared Bronski, Blake Barker, Vera Hur and Zhao Yang: *Stability for traveling fronts of the KdV-Burgers equation*
4:15–4:40	Atanas Stefanov: *Kinks of fractional φ^4 models: existence, uniqueness, monotonicity, and sharp asymptotics*
4:40–5:05	Jared Bronski, Vera Hur and Robert Marangell: *Floquet theory and stability analysis for Hamiltonian PDEs*
5:05–5:30	Milena Stanislavova: *Existence and stability for the travelling waves of the Benjamin equation*
5:30–5:55	Mathew Johnson, Brett Ehrman and Stephane Lafortune: *Orbital stability of smooth solitary waves in the Novikov equation*

3:50–5:55	**SESSION 6**, Ballroom A: Recent developments in nonlinear waves: From rogue waves and blow-ups to shocks, vortices and beyond - II/II
	Chairs: Efstathios Charalampidis, Ricardo Carretero-Gonzalez and Panayotis Kevrekidis
3:50–4:15	Sathyanarayanan Chandramouli, Simeon Mistakidis, Garyfallia Katsimiga and Panayotis Kevrekidis: *Shock waves in the extended Gross-Pitaevskii equation*
4:15–4:40	Jianke Yang: *Rogue curves in the Davey–Stewartson I equation*

4:40–5:05 Annalisa Calini and Thomas Ivey: *A novel geometric realization of the Yajima-Oikawa equations* (181)
5:05–5:30 Simeon Mistakidis and George Bougas: *Turbulence in driven dipolar gases across the superfluid-to-super solid phase transition*
5:30–5:55 Ziad Musslimani: *Collapse dynamics for two-dimensional space-time nonlocal nonlinear Schrodinger equations*

3:50–5:55 **SESSION 25**, Ballroom B: Emergent dynamics in integrable systems: soliton gasses and related topics - I/II, Chairs: Robert Jenkins and Alexander Tovbis
3:50–4:15 Guido Mazzuca, Manuela Girotti, Tamara Grava, Robert Jenkins, Ken McLaughlin and Maxim Yattselev: *Soliton trains with randomness*
4:15–4:40 Seung-Yeop Lee and Abril Arenas: *Planar orthogonal polynomials with non-Hele-Shaw type polynomial potentials*
4:40–5:05 Marco Bertola and Alexander Tovbis: *Minimal Dirichlet energy for fNLS condensates*
5:05–5:30 Alexander Tovbis and Marco Bertola: *Focusing NLS condensates of minimal averaged intensity*
5:30–5:55 Thibault Congy, Gennady El and Mark Hoefer: *Wave-mean field interaction in integrable turbulence*

3:50–5:55 **SESSION 27**, Ballroom C: Recent theoretical and numerical advances in fluids and nonlinear optics - II/II Chairs: Denis Silantyev, Sergey Dyachenko and Alexandr Chernyavsky
3:50–4:15 Barbara Prinari: *Local and global well-posedness for the Maxwell-Bloch equations with inhomogeneous broadening*
4:15–4:40 Yifeng Mao, Gino Biondini, Gennady El and Mark Hoefer: *Generalized rarefactions and dispersive shock waves for the Korteweg-de Vries equation*
4:40–5:05 Adilbek Kairzhan: *A Hamiltonian Dysthe equation for water waves*
5:05–5:30 Pavel M Lushnikov: *Exact solution and integrability of ballistic motion of fluid with free surface*
5:30–5:55 Denis Silantyev: *Computing nearly-extreme Stokes waves with high-precision using conformal maps*

3:50–5:55 **SESSION 12**, Ballroom D: Singular asymptotics for integrable nonlinear waves and related topics - II/III Chairs: Deniz Bilman, Robert Buckingham and Peter Miller
3:50–4:15 Harini Desiraju, Alexander Its and Andrei Prokhorov: *Nonlinear steepest descent on a torus: A case study of the Landau-Lifshitz equation*
4:15–4:40 Maksim Kosmakov: *Nonlinear steepest descent approach to the singular asymptotics of the radial Toda equation*
4:40–5:05 Giorgio Young, Deniz Bilman, Eliot Blackstone and Peter Miller: *Rational solutions to the mKdV equation*
5:05–5:30 Robert Buckingham, Robert Jenkins and Peter Miller: *Atypical focusing of semiclassical soliton ensembles in the AKNS hierarchy*
5:30–5:55 Chris Discenza and Peter Miller: *The semiclassical modified nonlinear Schrodinger equation with Talanov initial data*

7:00–9:00 **BANQUET**
Thiab Taha: Presentation of Students' Travel Awards

Wednesdsay, April 16, 2025

8:00–9:00 **KEYNOTE LECTURE III**, Athena Ballroom E
Peter Engels: *Ultracold quantum gases: A prime testbed for studying nonlinear dynamics*
Chair: Jerry Bona

9:10–10:00 **SESSION 8**, Athena Ballroom E: Geometric methods in spectral theory of traveling waves - IV/V
Chairs: Mitch Curran, Yuri Latushkin and Selim Sukhtaiev
9:10–9:35 Vahagn Manukian, Anna Ghazaryan, Jonathan Waldmann and Priscilla Yinzime: *Traveling front solutions in diffusive SIS model*
9:35–10:00 Emmanuel Fleurantin, Christopher Jones and Jeremy Marzuola: *Counting gap eigenvalues for the perturbed 3D nonlinear Schrödinger equation by a Maslov index* (10)

9:10–10:00 **SESSION 4**, Ballroom A: Numerical and theoretical solutions of wave and kinetic equations - I/II
Chair: Minh-Binh Tran
9:10–9:35 Christof Sparber: *Numerical evidence for singularity formation in defocusing fractional NLS in one space dimension*
9:35–10:00 Alejandro Aceves, Sabrina Hetzel and Ross Parker: *On the dynamics and generation of optical quartic solitons*

9:10–10:00 **SESSION 18**, Ballroom B: Existence and stability of traveling waves - II/II
Chairs: Anna Ghazaryan, Stephane Lafortune and Vahagn Manukian
9:10–9:35 Panayotis Kevrekidis: *Kinks in nonlinear Klein-Gordon models: from variable nonlinearity to fractional dispersion*
9:35–10:00 Brett Ehrman and Mathew Johnson: *Transverse spectral instability of the Novikov-KP equation*

9:10–10:00 **SESSION 24**, Ballroom C: Recent Advances in photonic systems - II/II
Chairs: Justin Cole and Ziad Musslimani
9:10–9:35 Troy Johnson: *Bulk and edge solitons of the nonlinear Haldane model*
9:35–10:00 Miguel A. Bandres, Bradford Geiger, Enmanuel Guzman and Konrad Tschernig: *Exploring new frontiers in branched flow of light*

9:10–10:00 **SESSION 23**, Ballroom D: Painlevé equations, integrable systems, and related topics - III/III
Chairs: Anton Dzhamay, Pieter Roffelsen and Alexander Stokes
9:10–9:35 Kanam Park: *Affine Weyl group actions on a 3×3 Lax form for the q-*$(E_6^{(1)})$ *Painlevé equation*
9:35–10:00 Alexander Stokes and Pieter Roffelsen: *Real and imaginary roots of generalised Okamoto polynomials and partial-rogue waves in the Sasa-Satsuma equation*

10:00–10:25 COFFEE BREAK

10:30–12:10 **SESSION 8**, Athena Ballroom E: Geometric methods in spectral theory of traveling waves - V/V
Chairs: Mitch Curran, Yuri Latushkin and Selim Sukhtaiev
10:30–10:55 Montie Avery: *Spectral criteria for front invasion speeds*
10:55–11:20 Alim Sukhtayev and Nathaniel Smith: *Splitting quantum graphs*
11:20–11:45 Samuel Walsh: *Finite-time self-similar implosion of hollow vortices*
11:45–12:10 Anna Ghazaryan, Stephane Lafortune, Yuri Latushkin and Vahagn Manukian: *On the spectrum of the front in a predator-prey model*

10:30–12:10 **SESSION 4**, Ballroom A: Numerical and theoretical solutions of wave and kinetic equations - II/II
Chair: Minh-Binh Tran
10:30–10:55 Thomas Chen: *Emergence of quantum Boltzmann dynamics in interacting boson and fermion gases*
10:55–11:20 Thomas Hagstrom: *Direct computation of singular solutions for fluid models*
11:20–11:45 Kui Ren, Lu Zhang and Yin Zhou: *An energy-based discontinuous Galerkin method for the nonlinear Schrodinger equation with wave operator*
11:45–12:10 Benno Rumpf: *Less is different: coherent structures make turbulence in one dimension difficult*

10:30–12:10 **SESSION 25**, Ballroom B: Emergent dynamics in integrable systems: soliton gasses and related topics - II/II, Chairs: Robert Jenkins and Alexander Tovbis
10:30–10:55 Elliot Blackstone, Peter Miller and Matthew Mitchell: *Universality in the small-dispersion limit of the Benjamin-Ono equation*
10:55–11:20 Abey Lopez-Garcia and Alexander Tovbis: *Non-standard Green energy problems in the complex plane*
11:20–11:45 Kurt Schmidt, Matt Mitchell and Robert Buckingham: *The small dispersion limit of KdV*

10:30–12:10 **SESSION 1**, Ballroom C: Nonlinear waves and applications - II/II
Chairs: Jerry Bona, Hongqiu Chen
10:30–10:55; Weiwei Hu: *Feedback control design for mixing in incompressible flows*
10:55–11:20 Min Chen: *Mathematical analysis of bump to bucket problem*
11:20–11:45 Pamela Guerrero: *A study of BBM type equations*
11:45–12:10 Fernando Lund: *Second harmonic generation of acoustic waves in a nonlinear elastic solid*

10:30–12:10 **SESSION 21**, Ballroom D: Advances in integrable systems and inverse scattering - IV/IV
Chairs: Barbara Prinari and Gino Biondini

10:30–10:55 Alexander Chernyavsky, Gino Biondini and John Ringland: *Whitham modulation theory for the Davey-Stewartson system*

12:10–1:40 LUNCH

1:40–3:20 **SESSION 12**, Athena Ballroom E: Singular asymptotics for integrable nonlinear waves and related topics - III/III, Chairs: Deniz Bilman, Robert Buckingham and Peter Miller

1:40–2:05 Anton Dzhamay, *Orthogonal polynomials and discrete Painlevé equations*

2:05–2:30 Thomas Trogdon: *On the asymptotics of Jacobi-type orthogonal polynomials on multiple intervals with non-analytic weights*

2:30–2:55 Trevor Johnson: *Large-parameter Asymptotics for rational solutions of Painlevé V*

2:55–3:20 Charbel Abi Younes and Tom Trogdon: *Asymptotics for orthogonal polynomials with applications in spectral density estimation*

1:40–3:20 **SESSION 20**, Ballroom B: Investigation of Nonlinear Dispersive wave systems - III/III
Chairs: Min Chen, Olivier Goubet and Bingyu Zhang

1:40–2:05 Bingyu Zhang: *Global well-posedness of the initial-boundary value problem of a class generalized KdV equation on a finite interval*

2:05–2:30 *Ivonne Rivas and Liliana Esquivel: L2 well-posedness and bounded controllability for the Korteweg-deVries-Burgers equation in a half-plane*

2:30–2:55 Lionel Rosier and Francisco Vielma: *Control of the intermediate long wave equation on a periodic domain*

2:55–3:20 Marcelo Moreira Cavalcanti, Valeria Neves Cavalcanti, Carole Rosier and Lionel Rosier: *Numerical control of a semilinear wave equation on an interval*

1:40–3:20 **SESSION 22**, Ballroom D: Nonlinear Waves in Lattices - II/II
Chairs: Christopher Chong and Timothy Faver

1:40–2:05 J. Douglas Wright: *Approximation of Calogero-Moser lattices by Benjamin-Ono equations*

2:05–2:30 Timothy Faver, J. Douglas Wright and Hermen Jan Hupkes: *Dimer FPUT periodics without symmetry*

2:30–2:55 Akpan, Udoh: *Solitary waves in FPUT Lattices with long range particle interactions*

2:55–3:20 Patrick Sprenger: *Dispersive hydrodynamics of a discrete conservation law*

1:40–3:20 **PAPERS**, Ballroom C, Chair: John Albert

1:40–2:05 Jerry Bona, Hongqiu Chen, Pamela Guerrero, Cristina Haidau and Sanja Pantic: *The initial-value problem for a Gardner-type equation*

2:05–2:30 John Albert and Jack Arbunich: *Stability and instability of bound states for a regularized NLS equation*

2:30–2:55 Sean Nixon: *Spiral waves and solitons in Floquet lattices*

1:40–2:55 **PAPERS**, Ballroom A, Chair: TBD

1:40–2:05 Iftikhar Ahmad, Hina Zahid, Hira Ilyas, Muhammad Asif Zahoor Raja and Amna Munir: *Artificial intelligence with Levenberg-Marquardt for calculating the thermal performance of convective radiative straight fin with the triangular shape*

2:05–2:30 Wuchen Li, *Numerical analysis on neural network projected schemes for approximating one-dimensional Wasserstein gradient flows*

2:30–2:55 Darren Crowdy: *Water waves with vorticity and the Schwarz function*

LIST OF ORGANIZED SESSIONS

1. *Nonlinear waves and applications* (Jerry Bona, Hongqiu Chen, Min Chen and Vassili Dougalis)
2. *Evolution equations and integrable systems* (Alex Himonas, Curtis Holliman, Fangchi Yan)
3. (Canceled)
4. *Numerical and theoretical solutions of wave and kinetic equations* (Binh Minh Tran)
5. *Peakons, kinkons and other weak solutions for nonlinear wave equations* (Stephen Anco, Zhijun (George) Qiao, Vesselin Vatchev)
6. *Recent developments in nonlinear waves: From rogue waves and blow-ups to shocks, vortices and beyond* (Stathis Charalampidis, Panos Kevrekidis, Ricardo Carretero)
7. *Recent and alternate methods for the numerical solution of partial differential equations* (Muhammad Usman, Chaudry Masood Khalique)
8. *Geometric methods in spectral theory of traveling waves* (Mitch Curran, Yuri Latushkin, Selim Sukhtaiev)
9. *Recent trends in acoustics and related wave phenomena: Computational and analytical methodologies* (James Lambers)
10. *Recent advances in nonlinear differential equations and applications* (Andrei Ludu, Harihar Khanal, Stefan A. Carstea)
11. *Water waves* (John Carter, Bernard Deconinck)
12. *Singular asymptotics for integrable nonlinear waves and related topics* (Deniz Bilman, Robert Buckingham, Peter Miller)
13. *Nonlinear partial differential equations and dynamical behavior of solutions* (Wei Lian, Runzhang Xu)
14. *Recent advances in algorithmic development for nonlinear PDEs* (Qi Wang, Jia Zhao, Xuelong Gu)
15. *Recent advances in stability of nonlinear waves* (Jared Bronski, Graham Cox, Robert Marangell)
16. *Recent developments on free surface flows* (Ming Chen, Samuel Walsh, Miles Wheeler)
17. *Fractional calculus and its applications* (Yang Liu)
18. *Existence and stability of traveling waves* (Anna Ghazaryan, Stephane Lafortune, Vahagn Manukian)
19. *Stability for nonlinear waves with singularities* (Geng Chen, Yannan Shen)
20. *Investigations of nonlinear dispersive wave systems* (Min Chen, Olivier Goubet and Bingyu Zhang)
21. *Advances in integrable systems and inverse scattering* (Gino Biondini, Barbara Prinari)
22. *Nonlinear waves in lattices* (Christopher Chong, Timothy Faver)
23. *Painlevé equations, integrable systems, and related topics* (Anton Dzhamay, Pieter Roffelsen, Alexander Stokes)
24. *Recent advances in photonic systems* (Justin Cole, Ziad Musslimani)
25. *Emergent dynamics in integrable systems: soliton gasses and related topics* (Alex Tovbis, Robert Jenkins)
26. *Waves in complex mathematical models of optics and superconductivity* (Jean-Guy Caputo, Alejandro Aceves)
27. *Recent theoretical and numerical advances in fluids and nonlinear optics* (Sergey Dyachenko, Denis Silantyev, Alexandr Chernyavsky)

KEYNOTE ABSTRACTS

Recent developments in dispersive hydrodynamics

Mark A. Hoefer
University of Colorado
hoefer@colorado.edu

Dispersive hydrodynamics encompass the study of multiscale nonlinear dispersive waves, with dispersive shock waves or DSWs a canonical class of solutions. Scale separation leads to the natural mathematical framework of Whitham modulation theory, a long wavelength description of nonlinear dispersive wavetrains. From modulations of multiphase or multidimensional nonlinear wavetrains to the launching of nonlinear waves from a boundary, this talk will present a variety of developments and results in dispersive hydrodynamics utilizing Whitham modulation theory, numerical simulations, and experiment.

Advancing stability through rigorous computation

Vera Mikyoung Hur
University of Illinois Urbana-Champaign
verahur@illinois.edu

Mathematical stability analysis has traditionally relied on a suite of well-established methodologies, such as the celebrated Grillakis–Shatah–Strauss criterion, which have successfully addressed numerous challenges and will remain invaluable. However, the field is ripe for fresh perspectives and innovative insights. Enter Computer-Assisted Proofs (CAP)—a cutting-edge technique capable of tackling problems once considered intractable. I will explore how CAP techniques can be applied to solve longstanding stability problems and, in the process, contribute to the development of CAP itself. I will demonstrate how to combine rigorous analysis, numerical computation, and CAP techniques, where CAP plays a pivotal role in rigorous enclosing solutions and validating stability criteria.

I will begin by discussing the stability of traveling front solutions of the KdV–Burgers equation. We have recently proved a nonlinear asymptotic stability result, which significantly improves upon previous work as the result neither relies on profile monotonicity nor small perturbations. We have leveraged CAP techniques to verify the stability criterion across the numerically-conjectured range of the parameter, encompassing genuinely oscillating profiles.

I will also address the stability of periodic traveling waves in non-integrable Hamiltonian PDEs, where a major challenge is establishing that the essential spectrum of the associated linearized operator is purely imaginary. This is in contrast to modulational instability results. For the generalized KdV equation, for example, where rigorous mathematical treatments have been limited to a few integrable systems, small-amplitude solutions, and the solitary wave limit, our ongoing efforts focus on establishing analytical bounds for potentially unstable eigenvalues and rigorously computing these eigenvalues across the entire range of the Floquet exponent.

Ultracold quantum gases: A prime testbed for studying nonlinear dynamics

Peter Engels
Washington State University
engels@wsu.edu

Ultracold quantum gases play a key role in modern physics. These systems consist of clouds of atoms that are cooled to temperatures near absolute zero using laser cooling and related techniques, making them some of the coldest entities in the universe. They offer direct experimental access to the quantum mechanical world, which describes nature's behavior at a fundamental level. While quantum mechanics is often associated with atomic-scale phenomena, ultracold quantum gases allow us to explore quantum behavior in objects approaching millimeter sizes, all while maintaining full quantum mechanical properties.

A hallmark of quantum mechanics is the wave-like behavior of atoms, leading to their description as matter waves. Under the right conditions, a large number of atoms can occupy a single, macroscopic matter wave, forming a Bose-Einstein condensate (BEC). Interatomic interactions induce nonlinearities in the system, resulting in rich dynamics. Consequently, BECs are central to advancing our understanding of quantum hydrodynamics.

From dispersive shocks and turbulence to vorticity, solitons, hydrodynamic instabilities, caustics and more, quantum hydrodynamics offers a vast playground for both experimental and theoretical investigations. At Washington State University, our group has a long history of experimenting with BECs, with a particular focus on studying solitons in elongated, channel-like geometries. We have explored both individual features, such as our recent realization of a Peregrine soliton, and large aggregates of solitons, including soliton-gas-like systems. Our work thrives through collaborations with applied mathematicians. The combination of experimental, analytical, and numerical studies has led to significant progress and promises further advancements. In this talk, I will introduce this highly active area of modern science and highlight recent results from our group.

SESSION ABSTRACTS

SESSION 1

Theory and application of nonlinear wave models

Jerry L. Bona
University of Illinois at Chicago
jbona@uic.edu

Hongqiu Chen
University of Memphis
hchen1@memphis.edu

Min Chen
Eastern Institute of Science and Technology, Ningbo, China
Purdue University
chen4544@yahoo.com

Vassili Dougalis
National and Kapodistrian University of Athens, Greece

This session will be mostly concerned with applications of nonlinear, dispersive wave models.

SESSION 2

Evolution equations and integrable systems

Alex Himonas
University of Notre Dame
himonas.1@nd.edu

Curtis Holliman
The Catholic University of America
holliman@cua.edu

Fangchi Yan
Virginia Tech
fangchi@vt.edu

Linear and nonlinear evolution equations have been at the forefront of advances in partial differential equations for a long time. They are involved in beautiful, yet extremely challenging problems, with a strong physical background, for which progress is achieved through a mixture of techniques lying at the interface between analysis and integrable systems. Topics studied for these equations include, among others, traveling waves, initial and boundary value problems, local and global well-posedness, inverse scattering, stability, and integrability.

SESSION 4

Numerical and theoretical solutions of wave and kinetic equations

Minh-Binh Tran
Texas A&M University
mailto:minhbinh@tamu.edu

The wave equation is a partial differential equation describing waves that includes traveling and standing waves. Kinetic equations describe the evolution of distribution function of molecules or other objects (like electrons, ions, stars, galaxy, or galactic aggregations) with respect to their velocities and space coordinates with respect to time. This session covers some selected talks concerning the progress in both fields.

SESSION 5

Peakons, kinkons, and other weak solutions for nonlinear wave equations

Stephen Anco
Brock University, Canada
sanco@brocku.ca

Zhijun (George) Qiao
University of Texas Rio Grande Valley
zhijun.qiao@utrgv.edu

Vesselin Vatchev
University of Texas Rio Grande Valley
vesselin.vatchev@utrgv.edu

Nonlinear wave phenomenon is one of very important components of applied mathematics and nonlinear sciences with profound relations to PDEs, Lie algebra, nonlinear analysis, symplectic geometry, and other branches of mathematics as well as significant applications in nonlinear optics, fluid dynamics, theoretical mechanics, theoretical physics and mathematical physics, and many other natural and social sciences. This session provides a platform for active researchers to present their latest work on nonlinear wave equations arising from mathematical physics with peakon, kinkon and other weak solutions, including Camassa-Holm equation, Degasperis-Procesi equation, Boussinesq equation, negative order flows of integrable hierarchy, and their related physical models. Our participants will share their recent results, discuss open challenges, and build new collaborative ties during the conference. We hope that this session can cast on a lasting impact on the future of the field and the research impact of the session topics will be significant for all our participants.

SESSION 6

Recent developments in nonlinear waves: From rogue waves and blow-ups to shocks, vortices and beyond

Efstathios G. Charalampidis
San Diego State University
echaralampidis@sdsu.edu

Ricardo Carretero-Gonzalez
San Diego State University
rcarretero@sdsu.edu

Panayotis G. Kevrekidis
University of Massachusetts Amherst
kevrekid@umass.edu

This session will focus on analytical, computational and experimental advances of several types of nonlinear waves in nonlinear dispersive evolutionary models. In particular, analytical and computational techniques for rogue and dispersive shock waves in discrete and continuum models will be introduced together with respective results that are relevant in optics, fluid mechanics, and ultracold physics applications. Moreover, recent results of studies revolving around vortical patterns as well as droplet structures in mean-field-type models and beyond will be discussed, and experimental links will be made accordingly. Advances in the analysis of and computational techniques for blow-up in nonlinear dispersive models will also be explored. This session aims to bring together experts and junior researchers in the respective fields, thus enabling the cross-polination of ideas and approaches for studying fundamental models of nonlinear waves.

SESSION 7

Recent and alternate methods for the numerical solution of partial differential equations

Muhammad Usman
University of Dayton
musman1@udayton.edu

Chaudry Masood Khalique
North-West University, Mafikeng Campus, South Africa
Masood.Khalique@nwu.ac.za

Nonlinear differential equations (ordinary, and partial) describe many physical phenomena arising in science and engineering. Thus, finding their solutions play a vital role in providing information to understand and interpret the structure of such physical phenomena. Researchers have developed many analytical and numerical methods to solve these equations. Recent numerical methods include finite difference methods, collocation methods, finite element methods and using artificial neural network(ANN). While testing numerical techniques, when exact solutions of initial and boundary value problem of nonlinear partial differential equations are not available, conservation laws play an important role. Well-known analytical tools include Lie symmetry method, Bäcklund transformation method, and inverse scattering transformation method.

This special session is dedicated to showcase recent progress in finding analytical and numerical solutions to nonlinear differential equations by various methods and to stimulate collaborative research activities.

SESSION 8

Geometric methods in spectral theory of traveling waves

Mitch Curran
Auburn University
mtc0076@auburn.edu

Yuri Latushkin
University of Missouri
latushkiny@missouri.edu

Selim Sukhtaiev
Auburn University
szs0266@auburn.edu

We will bring together researchers working on various stability issues for such special solutions of partial differential equations as periodic and solitary waves. All aspects of stability/instability will be discussed, with a special emphasis on methods of spectral theory that lie at the heart of stability analyses. It will be expected of speakers that they will spend some time explaining the perspective underlying their work in order to stimulate further discussion and collaboration in the field.

A particular main theme will be applications of infinite dimensional symplectic geometry in the spectral theory of the operators obtained by linearizing the partial differential equation about the traveling wave or other special solution. Specifically, we expect that some talks will be concerned with the relation of the Maslov index, a topological invariant defined as the signed number of intersections of a path formed by Lagrangian subspaces with a train of a fixed subspace, and the Morse index counting the number of unstable eigenvalues of the linearization. Recently this topic has been the focus of attention of a large group of researchers, and a special session with this emphasis will foster further collaborations in this area.

SESSION 9

Recent trends in acoustics and related wave phenomena: computational and analytical methodologies

James V. Lambers
University of Southern Mississippi
James.Lambers@usm.edu

The aim of this special session is to highlight recent research, both analytical and numerical, relating to acoustics, its sub-fields, and related fields that involve the study and utilization of wave phenomena. Topics presented include, but are not limited to, combinations of analytical and numerical methods for the solution of partial differential equations used to model wave propagation.

SESSION 10

Recent advances in nonlinear differential equations and applications

Andrei Ludu
Embry-Riddle Aeronautical University
ludua@erau.edu

Harihar Khanal
Embry-Riddle Aeronautical University
khanna66a@erau.edu

Stefan A. Carstea
National Institute of Physics and Nuclear Engineering, Romania
acarst@theory.nipne.ro

This session will focus on recent advances in modeling and computation of nonlinear systems, including analysis of field data and wave tank experiments. Nonlinear waves are ubiquitous in long and slender systems like channels, fiber optics, electric lines, blood arteries, nerves, but also occur spontaneously in world's oceans, littoral regions, around river mounds, and water systems with bathymetry. Nonlinear waves are also connected to mixing processes, transport mechanisms and ice redistribution. In the session there will be presentations on coupled nonlinear oscillators, liquid crystal systems and solitons in compact systems like liquid drops, nuclei or neutron stars. This session will feature a blend of analytical, numerical and experimental work. Among the topics to be discussed here are numerical and experimental studies of solitary waves.

SESSION 11

Water waves

John D. Carter
Seattle University
carterj1@seattleu.edu

Bernard Deconinck
University of Washington
deconinc@uw.edu

This minisymposium focuses on nonlinear water waves. The speakers will present analytical and numerical results from mathematical models of waves on shallow and/or deep water.

SESSION 12

Singular asymptotics for integrable nonlinear waves and related topics

Deniz Bilman
University of Cincinnati
bilman@ucmail.uc.edu
Robert Buckingham
University of Cincinnati
buckinrt@ucmail.uc.edu
Peter Miller
University of Michigan
millerpd@umich.edu

This session features talks on various problems in integrable systems theory, especially applied to nonlinear wave propagation problems. Most of the talks relate to phenomena described by singular asymptotics.

SESSION 13

Nonlinear partial differential equations and dynamical behavior of solutions

Wei Lian
Lund University, Sweden
wei.lian@math.lu.se, lianwei_1993@163.com
Runzhang Xu
Harbin Engineering University, China
xurunzh@163.com

The session will present some recent advances in mathematical analysis of nonlinear partial differential equations and dynamical behaviors of solutions. Topics include global quantitative stability of wave equation, well-posedness theories of nonlinear evolution systems, and some related stability and existence results to PDEs.

SESSION 14

Recent advances in algorithmic development for nonlinear PDEs

Jia Zhao
University of Alabama
jia.zhao@ua.edu
Xuelong Gu
University of South Carolina
XGU@mailbox.sc.edu
Qi Wang
University of South Carolina
qwang@math.sc.edu

This special session will focus on recent advances in algorithmic development for nonlinear PDEs, aiming to foster collaboration in computational methods for solving nonlinear PDEs in complex nonlinear wave phenomena. We will bring together researchers working on cutting-edge numerical methods and computational techniques. Some topics will include, but are not limited to, structure-preserving algorithms, physics-informed numerical methods, model reduction techniques, and high-order schemes, emphasizing theoretical insights and applications.

SESSION 15

Recent advances in stability of nonlinear waves

Jared Bronski
University of Illinois at Urbana–Champaign
bronski@illinois.edu
Graham Cox
Memorial University, Canada
gcox@mun.ca
Robert Marangell
University of Sydney, Australia
robert.marangell@sydney.edu.au

In the study of nonlinear waves and coherent structures questions of stability are of paramount importance. The details of the nature of the spectral problem are often deeply informed by underlying structures, such as the Hamiltonian structure. This mini symposium brings together researchers working on a diverse set of problems which exhibit some underlying structure which can be used to shed light on the question of the stability of nonlinear waves or other coherent solutions.

SESSION 16

Recent developments on free surface flows

Robin Ming Chen
University of Pittsburgh
mingchen@pitt.edu
Samuel Walsh
University of Missouri
walshsa@missouri.edu
Miles H. Wheeler
University of Bath, United Kingdom
mw2319@bath.ac.uk

This session will explore recent advances in the study of free surface flows, with an emphasis on rigorous mathematical arguments and explicit analytical solutions. Major themes include the existence, stability, and qualitative theory of traveling waves in water as well as the dynamics of hollow vortices and bubbles.

SESSION 17

Fractional calculus and its applications

Yang Liu
Inner Mongolia University, China
mathliuyang@imu.edu.cn

In recent years, a significant increase in research efforts across various scientific and engineering fields has been focused on dynamical systems governed by fractional differential equations (FDEs). Numerous computational fractional dynamic systems and their applications have emerged, signifying the escalating significance of this field. The aims of the Session are to foster communication among researchers and practitioners who are interested in this field, introduce new researchers to the field, showcase innovative concepts, report state-of-the-art and in-progress research results, discuss future trends and challenges, and facilitate collaborations across disciplines. Topics include, but are not limited to:

(i) Mathematical modelling of fractional dynamic systems, analytical and numerical techniques to solve these systems.

(ii) Numerical methods and numerical analysis, e.g., finite difference method, finite element method, finite volume method, spectral method, matrix method, meshless method, etc.

(iii) Applications of fractional dynamic systems in electromagnetics, biology, signal and image processing, environmental science, finance, fluid mechanics, chemistry, physics, etc.

SESSION 18

Existence and stability of traveling waves

Anna Ghazaryan
Miami University
ghazarar@miamioh.edu

Stéphane Lafortune
College of Charleston
LafortuneS@cofc.edu

Vahagn Manukian
Miami University
manukive@miamioh.edu

Traveling waves are key structures that shed light on pattern formation mechanisms in spatially extended nonlinear systems. Speakers in this session will discuss their new results about structural properties of coherent structures in nonlinear models, their stability and instability properties and variety of bifurcations scenarios.

SESSION 19

Stability for nonlinear waves with singularities

Geng Chen
University of Kansas
gengchen@ku.edu

Yannan Shen
University of Kansas
yshen@ku.edu

In this section, we focus on the stability of solutions for various physical models from nonlinear waves, fluid dynamics and liquid crystals. The solutions of these models often form singularities. The wellposedness and behavior of solution, especially the stability, describe very interesting and challenging problems.

SESSION 20

Investigation of nonlinear dispersive wave systems

Min Chen
Eastern Institute of Technology, China
chen45@purdue.edu

Olivier Goubet
Laboratoire Paul Painlevé,, U-Lille and INRIA, France
olivier.goubet@univ-lille.fr

Bingyu Zhang
University of Cincinnati
zhangb@ucmail.uc.edu

This session will be centered around the investigation of nonlinear dispersive wave systems. Features will be theoretical investigation, such as global and local wellposedness and stability on various equations (such as Boussinesq system for internal waves, surface electromigration equations, Majda-Biello systems) with different boundary conditions (such as Cauchy problem, finite interval and half line). and numerical investigations on equations with nonlocal viscous dissipative terms, control of a semilinear wave equations. The results on investigation on Schrödinger-Poisson equation with doping profile, modeling of forest ecosystem and comparison principles will also be presented in this session.

SESSION 21

Advances in integrable systems and inverse scattering

Gino Biondini
State University of New York Buffalo
biondini@buffalo.edu

Barbara Prinari
State University of New York Buffalo
bprinari@buffalo.edu

Integrable partial differential equations are Hamiltonian systems that admit an infinite number of conserved quantities, and whose initial-value problem can be linearized via a method called the inverse scattering transform. The comprehensive understanding of integrable nonlinear wave equations, their solutions, and the investigation of their mathematical structures require the application of various techniques from diverse branches of mathematics, including exact methods, approximations, asymptotics, perturbation techniques, symmetry analysis, and numerical simulations, among others. The study of nonlinear waves and integrable systems has seen a marked revival in recent years, with growing connections to many fields of pure and applied mathematics, including multi-dimensional systems, boundary value problems, discretization issues, connections with algebraic and differential geometry, number theory and different areas of mathematics, etc. The proposed session aims at bringing together leading researchers in the fields of integrable partial differential equations, and at offering a broad overview of some of the current research activities at the frontier of the discipline.

SESSION 22

Nonlinear waves in lattices

Christopher Chong
Bowdoin College
cchong@bowdoin.edu

Timothy Faver
Kennesaw State University
tfaver1@kennesaw.edu

Lattices consisting of discrete particles coupled together by springs are paradigmatic models for wave propagation in a variety of discrete media. This session brings together junior and senior researchers alike to present on the latest work within this rich field. Analytical, numerical, and experimental approaches will be used to explore a variety of wave forms and other coherent structures such as traveling waves, breathers, dispersive shock waves and more.

This session is dedicated to the late Michael Herrmann. Professor Herrmann studied mathematics at Humboldt University in Berlin, earning his PhD there in 2005. After holding positions in Saarland and Muenster, he became a full professor of Mathematics at the TU Braunschweig, Germany, where he worked until his unexpected and much too early passing. His work in Mathematical Modeling, Applied Analysis, and Partial Differential Equations has gained international recognition, such as his work on modulation theory and phase transitions in systems with hysteresis. He loved immersing himself in new subjects, bringing fresh perspectives to a project, and was therefore a highly valued collaborator. Those who knew him admired his work, but also his general intellectual prowess and above all, his great kindness and humor.

SESSION 23

Painlevé equations, integrable systems, and related topics

Anton Dzhamay
Beijing Institute of Mathematical Sciences and Applications, China
adzham@bimsa.cn

Pieter Roffelsen
University of Sydney, Australia
pieter.roffelsen@sydney.edu.au

Alexander Stokes
Waseda University, Japan
stokes@aoni.waseda.jp

This session will focus on Painlevé equations as well as integrable systems more broadly, bringing together researchers working on geometric, algebraic and analytic aspects. The theory of Painlevé equations and their discrete analogues has reached a certain level of maturity, so one of the aims of this session is to explore future directions. There will be an emphasis on potential applications of these approaches and interplay with other areas, both within integrable systems and beyond. This session hopes to facilitate exchange between specialists in aspects of Painlevé equations and integrable systems (monodromy, Riemann-Hilbert theory, affine Weyl groups, geometric and representation-theoretic approaches) as well as related topics (orthogonal polynomials, random matrices, mathematical physics) to open up new connections.

SESSION 24

Recent advances in photonic systems

Justin Cole
University of Colorado, Colorado Springs
jcole13@uccs.edu

Ziad Musslimani
Florida State University
musliman@math.fsu.edu

New trends in the modeling, analysis, and application of electromagnetic waves in photonic media will be considered. Topics considered include, but are not limited to, topological insulators, solitons, and optical waveguides. Theoretical, experimental, and computational techniques are expected to be discussed.

SESSION 25

Emergent dynamics in integrable systems: soliton gasses and related topics

Robert Jenkins
University of Central Florida
robert.jenkins@ucf.edu

Alexander Tovbis
University of Central Florida
alexander.tovbis@ucf.edu

Integrable systems serve as fundamental models of dynamics in many areas of modern mathematics and physics. Integrability provides methods for precisely describing large families of solutions of these fundamental models. That said, in many situations of practical interest one is interested in systems composed of huge numbers of elementary components (many soliton problems, high genus finite-gap limits, etc.) In such situations, one wants to develop methods to describe the macroscopic dynamics of these many-body systems. This is at the heart of many challenging problems in integrable dynamics ranging from small-dispersion limits of PDEs, to problems in random matrices and integrable probability. In particular, the phenomena of emergence, is at the center of the theory of soliton gasses, an area of active research that has produced many interesting results in recent times. This session will bring together experts in integrable systems and related fields to discuss recent progress in the area and identify the new and challenging problems for the future.

SESSION 26

Waves in complex mathematical models of optics and superconductivity

Alejandro Aceves
Southern Methodist University
aaceves@mail.smu.edu

Jean-Guy Caputo
INSA Rouen Normandie, France
jean-guy.caputo@insa-rouen.fr

Superconducting materials have made great advances in recent years, driven by applications like field screening, levitation and superconducting electronics based on Josephson junctions. Optics has also seen interesting developments like topological photonics. In these fields, systems are engineered to obtain specific dynamic properties like for example increase the trapped flux of a superconductor, accelerate a kink in a Josephson junction, obtain a specific dispersion relation by arranging optical waveguides, etc..

This leads to complex mathematical models and interesting waves. This mini symposium will present a number of these models, their careful numerical treatment and their solutions.

SESSION 27

Recent theoretical and numerical advances in fluids and nonlinear optics

Denis Silantyev
University of Colorado at Colorado Springs
dsilanty@uccs.edu

Sergey Dyachenko
University at Buffalo
sergeydy@buffalo.edu

Alexandr Chernyavsky
Texas A&M Corpus Christi
alexandr.chernyavskiy@tamucc.edu

The session is devoted to new results found in both theoretical and numerical studies of fluid motion and nonlinear optical systems. Our session will examine new connections between special solutions of PDEs and their role in transition to turbulence, singularity formation. Various aspects of integrable systems and their solutions will be the subject of discussion in this session.

PAPER ABSTRACTS

Abstracts are listed in the order they appear in the schedule.

SESSION 1

Nonlinear waves and applications

The long wavelength limit of periodic solutions of water wave models

Hongqiu Chen
University of Memphis
hchen1@memphis.edu

It has been common practice for decades to approximate localized solutions of evolution equations set on unbounded spatial domains by solutions of associated periodic initial-value problems. No doubt that this practice goes back further, though not much further as computers of large enough scale to handle even one-space and one-time dimension problems were not available much earlier. Despite this method's long history, rigorous theory with error estimates is sparse. It is our purpose to provide theory with error estimates here. This is done in the context of long-crested, surface water wave models. Starting with *RLW-BBM* model, which is a low-level approximation for long-crested waves on the surface of water which propagates unidirectionally, then continue to higher-order model equations.

Minimum-uncertainty squeezed states for a time-dependent Schrödinger equation

Jose M. Vega-Guzman
Lamar University
jvegaguzman@lamar.edu

We elaborate on a class of solutions to the time-dependent Schrödinger equation for the simple harmonic oscillator in one dimension. They are derived by the action of the corresponding maximal kinematical invariance group on the standard ground state solution. The solution method has been used to construct soliton-like solutions of a nonlinear evolution equation. We show that the product of the variances attains the required minimum value only at the instances that one variance is a minimum and the other is a maximum, when the squeezing of one of the variances occurs. The generalized coherent states are explicitly constructed and their Wigner function is studied. The overlap coefficients between the squeezed, or generalized harmonic, and the Fock states are explicitly evaluated in terms of hypergeometric functions and the corresponding photon statistics are discussed. Some applications to quantum optics, cavity quantum electrodynamics and superfocusing in channelling scattering are mentioned. Explicit solutions of the Heisenberg equations for radiation field operators with squeezing are found.

S.I. Kryuchkov, S.K. Suslov, & J.M. Vega-Guzman, The minimum-uncertainty squeezed states for atoms and photons in a cavity, *J. Phys. B: Atomic, Molecular and Optical Physics.*, **46**(10) (2013), 104007.

Emergence of peaked singularities in the Euler-Poisson system

Junsik Bae
Korea Advanced Institute of Science and Technology, Korea
junsikbae@kaist.ac.kr
Sang-Hyuck Moon
Ulsan National Institute of Science and Technology, Korea
shmoon0208@unist.ac.kr
Kwan Woo
Seoul National University, Korea
kwanwoo@snu.ac.kr

We consider the one-dimensional Euler-Poisson system equipped with the Boltzmann relation. It is a fundamental fluid model which describes the dynamics of ions in an electrostatic plasma. Among others, the emergence of solitary waves is one of the most interesting phenomena in the dynamics of plasmas.

We provide the exact asymptotic behavior of the peaked solitary wave solutions near the peak. This enables us to study the cold ion limit of the peaked solitary waves with the sharp range of the Hölder exponents. Furthermore, we provide numerical evidence for C^1 blow-up solutions to the pressureless Euler-Poisson system, whose blow-up profiles are asymptotically similar to its peaked solitary waves (peakons). These solutions exhibit a different form of blow-up compared to the Burgers-type (shock-like) blow-up.

J. Bae, SH. Moon and K. Woo, Emergence of Peaked Singularities in the Euler-Poisson System. *J. Nonlinear Sci.*, **35**, 25 (2025)

Stability of stationary waves for viscoelastic fluids in half-space

Yoshihiro Ueda
Kobe University, Japan
ueda@maritime.kobe-u.ac.jp

In this talk, we discuss the stability of the compressible fluid with viscoelasticity. We consider the outflow problem in a one-dimensional half-space and show the existence of a stationary solution and its stability. There exists a lot of known results for compressible fluids. In particular, the existence and stability of stationary solutions to the outflow problem were discussed in Nakamura–Nishibata–Yuge [1] and Nakamura–Ueda–Kawashima [2], where the Mach number was used as a criterion. Similar results are obtained for viscoelastic fluids, however, the main feature is that the criterion is constructed by the modified Mach number, which takes into account the effect of viscoelasticity. This result is based on joint research with Yusuke Ishigaki of Osaka University.

1. T. Nakamura, S. Nishibata and T.Yuge, Convergence rate of solutions toward stationary solutions to the compressible Navier-Stokes equations in a half line, *J. Differential Equations*, **241** (2007), 94–111.
2. T. Nakamura, Y. Ueda and S. Kawashima, Convergence rate toward degenerate stationary wave for compressible viscous gases, *Nonlinear analysis and convex analysis*, Yokohama Publ., Yokohama, (2010), 239–248.

Feedback control design for mixing in incompressible flows

Weiwei Hu
University of Georgia
Weiwei.Hu@uga.edu

Understanding mass transport, fluid mixing, and their asymptotic behaviors via active control of the flow advection leads to fundamental, yet highly challenging problems often found in industrial and engineering applications. In this talk, we focus on nonlinear feedback control design for enhancing transport and mixing in incompressible flows. We will present some recent progresses as well as some open questions.

Mathematical analysis of bump to bucket problem

Min Chen
Eastern Advanced Institute of Science, China
chen4544@gmail.edu

In numerical simulations of surface water waves, when there is a deformation on the bottom, it is a common practice to transform form the boundary deformation data to the free surface. In this talk, we investigate this procedure, by comparing the waves generated by the moving bottom (Bump) and by the initial surface variation (Bucket), using linear and nonlinear Boussinesq-type models. This is a joint work with Olivier Goubet and Shenghao Li.

A study of BBM type equations

Pamela Guerrero
University of Memphis
pgrrero3@memphis.edu

The Benjamin Bona Mahony (BBM) equation was presented as an alternative to the Korteweg de Vries (KdV) equation in 1972. I begin by studying the modified-BBM equation initial value problem and work up to more general BBM type equations, each time changing the order and/or number of non-linear terms. Various techniques to show a priori bounds, depending on the regularity of the initial condition are discussed.

Second harmonic generation of acoustic waves in a nonlinear elastic solid

Fernando Lund
Universidad de Chile
flund@dfi.uchile.cl

Nonlinear ultrasound higher harmonics have become increasingly useful as a nonintrusive probe of both microstructure as well as damage of solid materials [1]. The current theoretical underpinning of these efforts rely on a formula for the second harmonic that is proportional to the square of the linear wave and grows linearly with distance away from the source [2]. This formula holds only for small distances, since otherwise there would be a violation of the conservation of energy. This restriction is here lifted.

Consider the one-dimensional problem of wave propagation in a weakly nonlinear elastic solid with a time harmonic loading of frequency localized at the origin, for a displacement u(x,t) as a function of position x and time t on the whole real line. Using a successive approximation scheme starting with the linear case, and a multiple time analysis, an explicit formula is found for the second harmonic that is valid for all x [1]. The second harmonic so obtained is an amplitude-modulated wave. For small distances from the source, the usual formula found in the literature [2] is recovered.

Consequences of this result for non-destructive testing will be discussed.

1. F. Lund, Second and third harmonic generation of acoustic waves in a nonlinear elastic solid in one space dimension, *J. Sound Vib.* **600** (2025) 118895, https://doi.org/10.1016/j.jsv.2024.118895 .
2. C. M. Kube et al., A unifying model of weakly nonlinear elastic waves; large on large theory, *J. Acoust. Soc. Am.* **151** (2022) 1294–1310, http://dx.doi.org/10.1121/10.0009101.

SESSION 2

Evolution equations and integrable systems

Initial-boundary value problems for NLS equations

A. Alexandrou Himonas
University of Notre Dame
himonas.1@nd.edu
Fangchi Yan
Virginia Polytechnic Institute and State University
fyan1@alumni.nd.edu

In this talk we shall discuss the well-posedness of initial-boundary value problems for nonlinear Schrödinger equations with cubic and quadratic nonlinearities. We will begin by solving the corresponding forced linear problem via the Fokas method and describing the linear estimates one can obtain when the forcing is in Bourgain spaces, the initial data in Sobolev spaces, and the boundary data in spaces that reflect boundary regularity. Then, we shall continue by describing the multilinear estimates which are suggested by norms of the forcing and which are used for proving that the iteration map defined by the Fokas solution formula is a contraction. Finally, we shall state the well-posedness results we obtain using our approach.

1. A. Himonas and F. Yan, *The Schrödinger equation with cubic nonlinearities on the half-line in low regularity spaces.* J. Math. Anal. Appl. **537** (2024), no. 1, Paper No. 128259.
2. A. Himonas and F. Yan, *A higher order quadratic NLS equation on the half-line*. J. Evol. Equ. (2024), to appear.
3. J.L. Bona, S.M. Sun and B.-Y. Zhang, *Nonhomogeneous boundary value problems of one-dimensional nonlinear Schrödinger equations.* J. Math. Pures Appl. **109** (2018), 1–66.
4. J. Holmer, *The initial-boundary-value problem for the 1D nonlinear Schrödinger equation on the half-line*. Differential Integral Equations **18** (2005), no. 6, 647–668.

A higher order quadratic NLS equation on the half-line

A. Alexandrou Himonas
University of Notre Dame
himonas.1@nd.edu
Fangchi Yan
Virginia Polytechnic Institute and State University
fyan1@alumni.nd.edu

The well-posedness of the initial-boundary value problem for higher order quadratic nonlinear Schrödinger equations on the half-line is studied by utilizing the Fokas solution formula for the corresponding linear problem. Using this formula, linear estimates are derived in Bourgain spaces for initial data in spatial Sobolev spaces on the half-line and boundary data in temporal Sobolev spaces suggested by the time regularity of the linear initial value problem. Then, the needed bilinear estimates are derived and used for showing that the iteration map defined via the Fokas solution formula is a contraction in appropriate solution spaces. Finally, well-posedness is established for optimal Sobolev exponents in a way analogous to the case of the initial value problem on the whole line.

1. A. Himonas and F. Yan, *A higher order quadratic NLS equation on the half-line*. J. Evol. Equ. (2024), to appear.
2. A. Himonas and F. Yan, *A higher dispersion KdV equation on the half-line*. J. Differential Equations **333** (2022), 55–102.

3. C.E. Kenig, G. Ponce, and L. Vega, *Quadratic forms for the 1-D semilinear Schrödinger equation.* Trans. Amer. Math. Soc. **348** (1996), no. 8, 3323–3353.

The linear BBM-equation on the quarter-plane, revisited: A novel rigorous approach and unexpected phenomena

Andreas Chatziafratis
National and Kapodistrian University of Athens, Greece
chatziafrati@math.uoa.gr
Jerry L. Bona
University of Illinois at Chicago
Hongqiu Chen
University of Memphis
Spyridon Kamvissis
University of Crete, Greece

We shall discuss some of our recent findings concerning the rigorous solution and analysis of fully non-homogeneous initial-boundary-value problems for the linearized Benjamin-Bona-Mahony equation on the spatiotemporal quarter-plane. The approach is based on complex-analytic tools and rigorous implementation of the Fokas unified transform method. Explicit solution formulae for the forced linear problem are thus derived in terms of contour integrals and analyzed for quite general initial values and boundary conditions in classical function spaces. A posteriori verification of the closed-form representations brings to the fore a single compatibility condition that must be obeyed by smooth data for a well-defined global solution, thereby indicating a type of boundary smoothing effect. Additional surprising observations (e.g., asymptotic instabilities) will be highlighted. For instance, both for Dirichlet and for Neumann boundary conditions, asymptotic periodicity holds. However, for Robin boundary conditions, we find not only that solutions lack the asymptotic periodicity property, but they in fact display instability, growing in amplitude exponentially in time. This is joint work with Jerry L. Bona, Hongqiu Chen and Spyridon Kamvissis [1].

1. Bona, J.L., Chatziafratis, A., Chen, H., Kamvissis, S. The linear BBM-equation on the half-line, revisited. *Lett. Math. Phys.* **114** (2024).

Illposedness of KdV-type equations using the modified energy method

Timur Akhunov
Wabash College
akhunovt@wabash.edu
Àngel Martìnez
Colegio Universitario Estudios Financieros, Spain
angeld.martinez@cunef.edu

Korteweg and de Vries in 1890s derived an equation that bears their name to elucidate unusual behavior of water waves. They discovered solitons that behave like billiard balls when interacting.
Can solitons be made compactly supported? Rosenau-Hyman in ’93 proposed a family of partial differential equations with such solutions that they dubbed “compactons”. Unlike KdV, wellposedness of compacton equations in standard Sobolev spaces is poorly understood. What conditions lead to illposedness? We share our recent results and conjectures on the topic using the modified energy method.
In particular, we demonstrate illposedness for an initial value for a fully nonlinear KdV type equation

$$\partial_t u = f(u_{xxx}, u_{xx}, u_x, u, x, t)$$

in high regularity Sobolev spaces. This result refines earlier illposedness results [1] and is sharp due to [2].

1. Didier Pilod, *On the cauchy problem for higher-order nonlinear dispersive equations*, Journal of Differential Equations **245** (2008), no. 8, 2055 – 2077.
2. Timur Akhunov, David M Ambrose, and J Douglas Wright, *Well-posedness of fully nonlinear KdV-type evolution equations*, Nonlinearity **32** (2019), no. 8, 2914–2954.

Current issues regarding the modeling of glioblastomas

Curtis Holliman
The Catholic University of America
holliman@cua.edu
Francesca Cozzi
University of Cambridge, UK
fmg39@cam.ac.uk

We examine some current issues regarding the modeling of progression and recurrence glioblastomas in a patient population. In the literature, the Fisher-Kolmogorov equation has been used to model the time-evolution of this cancer by means of a diffusion process, and we discuss how accurately this models actual tumor progression in addition to some new developments.

1. KR Swanson, RC Rostomily and EC Alvord Jr, A mathematical modelling tool for predicting survival of individual patients following resection of glioblastoma: a proof of principle, *British Journal of Cancer*, **98** (2008), 113 – 119.

Global well-posedness and scattering results for nonlinear wave equations

Guher Camliyurt
Virginia Tech
gcamliyurt@vt.edu
Carlos E. Kenig
University of Chicago
cek@math.uchicago.edu

In this talk, we will discuss the long-time behavior of large solutions to nonlinear wave equations in the energy-supercritical setting. We will review the concentration compactness and rigidity arguments through the seminal works of Kenig-Merle and Duyckaerts-Kenig-Merle for the energy-critical and energy-supercritical problems in three dimensions. We will then discuss the generalization of these methods to higher odd dimensions and the further challenges in even dimensions.

1. G. Camliyurt and C. E. Kenig, Scattering for radial bounded solutions of focusing supercritical wave equations in odd dimensions, *Nonlinear Anal.*, Paper No: 113352 (2023).
2. C. Kenig and F. Merle, Global well-posedness, scattering and blow-up for the energy critical focusing nonlinear wave equation, *Acta Math.*, **201(2)**, (2008), 147-212.

Nonlinear diffusion equations on the half line

John Holmes
Ohio State University
holmes.782@osu.edu

We investigate the well-posedness of the viscous Burgers' equation on the half-line using Fokas' novel unified transform method. We show that the Cauchy problem is well posed for initial data $u_0 \in H^s_x(0,\infty)$ and boundary data $g \in H_t^{\frac{2s+1}{4}}(0,T)$ when $1/2 < s < 0$. In particular, the solution $u \in C([0,T]; H^s_x(0,\infty)) \cap L^\infty_x((0,\infty); H_t^{\frac{2s+1}{4}}(0,T))$. We establish this result by finding the correct subsets of $H^s_x(0,\infty)$ and $H_t^{\frac{2s+1}{4}}(0,T)$ for which we can show appropriate linear and bi-linear estimates, and then apply the contraction mapping theorem for the solution operator constructed via the unified transform method. Our proof also shows that the solution, u, has the same regularity in the time variable as the boundary data, and is slightly smoother in the spatial variable then the initial data. It is known that index $s = 1/2$ is critical for the whole line problem; the Cauchy problem on the line is ill-posed for initial data below this index and well-posed for initial data at or above this index.

SESSION 4

Numerical and theoretical solutions of wave and kinetic equations

Numerical evidence for singularity formation in defocusing fractional NLS in one space dimension

Christof Sparber
University of Illinois at Chicago
sparber@uic.edu
Christian Klein
Institut de Mathématiques de Bourgogne, Institut Universitaire de France
christian.klein@u-bourgogne.fr

We consider nonlinear dispersive equations of Schrödinger-type involving fractional powers $0 < s \leq 1$ of the Laplacian and a defocusing power-law nonlinearity. Using a Fourier-spectral method, we conduct numerical simulations in the case of small, energy supercritical values of s and provide numerical evidence for a highly oscillatory singularity within the solution. This phenomenon seems to be similar to a recent analytical blow-up result by Merle et. al concerning defocusing energy supercritical NLS spatial dimensions higher than 5, see [1].

1. F. Merle, P. Raphaël, I. Rodnianski, and J. Szeftel, On blow up for the energy super critical defocusing nonlinear Schrödinger equations, *Invent. math.*, **227** (2022), 247-413.

On the dynamics and generation of optical quartic solitons

Sabrina Hetzel
Virginia Tech
shetzel9@vt.edu
Alejandro Aceves
Southern Methodist University
aaceves@smu.edu
Ross Parker
Center for Communications Research, Princeton
rosshparker@gmail.com

With the ability to engineer dispersion in an optical cavity, quartic solitons have emerged as viable alternative to classical solitons for different optical applications. This work presents theoretical and numerical studies on the generation and interactions of single and multiple pulses when the dispersion is of fourth order under different realizations.

Our results are based on a mean field approximation of a laser cavity leading to the modified Lugiato-Lefever equation (LLE). In particular we show that by constructing a double pulse initial condition for the LLE with quartic dispersion, we are able to find a discrete set of double pulse solutions whose stability alternates as the pulse separation distance increases.

Emergence of quantum Boltzmann dynamics in interacting boson and fermion gases

Thomas Chen
Department of MathematicsUniversity of Texas at Autin
tc@math.utexas.edu

We present some recent results regarding the emergence of quantum Boltzmann dynamics in interacting quantum manybody systems. For bosons, this phenomenon is studied in joint work with Michael Hott for the renormalized fluctuation field around a Bose-Einstein condensate. In the case of fermions, we present recent joint work with Esteban Cardenas which describes a subtle interplay between Boltzmann type collisions and interactions with bosonized electron-hole pairs.

1. E. Cardenas, T. Chen, Quantum Boltzmann dynamics and bosonized particle-hole interactions in fermion gases, https://arxiv.org/abs/2306.03300
2. T. Chen and M. Hott, On the emergence of quantum Boltzmann fluctuation dynamics near a Bose-Einstein condensate, J. Stat. Phys., **190** (4), 85 (2023).
3. T. Chen and M. Hott, Derivation of renormalized Hartree-Fock-Bogoliubov and quantum Boltzmann equations in an interacting Bose gas, https://arxiv.org/abs/2401.06298

Direct computation of singular solutions for fluid models

Thomas Hagstrom
Southern Methodist University
thagstrom@smu.edu

Inviscid fluid models often develop interesting singularities such as shocks and (possibly) vorticity blowups. As the theory in multiple space dimensions is incomplete, it is natural to use numerical simulations to investigate the development of singularities from smooth data. However, the direct use of standard discretizations often cannot reproduce the singularity formation with convincing convergence tests as t approaches the potential singularity time. Here we pursue a different strategy which can produce convergent approximations up to and through the singularity development and might even be able to compute singular solutions which do not evolve from an open set of initial conditions. Our approach is to first find a time-dependent transformation of the coordinates and fields so that the mapped solution remains smooth at the singularity time. We then evolve numerically the mapped system up to the singular point and achieve convergence with mesh refinement.

As our main example, we examine shock development in multiple space dimensions for the compressible Euler equations, for which the mathematical analysis still seems incomplete. Here we will compute shock development directly, using Alinhac's geometry-based formulation [1]. We will also speculate on the construction of singular solutions for the incompressible Euler equations using the vortex line coordinates of Kuznetsov and collaborators [2].

Acknowledgment: This work was supported in part by NSF Grant DMS-2309687. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the NSF.

1. S. Alinhac, A minicourse on global existence and blowup of classical solutions to quasilinear wave equations, *Journ'ees "Equations aux Derivées Partielles"* Forges-les-Eaux (2002).
2. E. A. Kuznetsov, Mixed Lagrangian-Eulerian description of vortical flows, *J. Fluid Mech.*, **600**, (2008), 167-180.

An energy-based discontinuous Galerkin method for the nonlinear Schrödinger equation with wave operator

Kui Ren
Columbia University
kr2002@columbia.edu
Lu Zhang
Rice University
lz82@rice.edu
Yin Zhou
Columbia University
yz3888@columbia.edu

This work develops an energy-based discontinuous Galerkin (EDG) method for the nonlinear Schrödinger equation with the wave operator. The focus of the study is on the energy-conserving or energy-dissipating behavior of the method with some simple mesh-independent numerical fluxes we designed. We establish error estimates in the energy norm that require careful selection of a weak formulation for the auxiliary equation involving the time derivative of the displacement variable. A critical part of the convergence analysis is to establish the L^2 error bounds for the time derivative of the approximation error in the displacement variable by using the equation that determines its mean value. Using a special weak formulation, we show that one can create a linear system for the time evolution of the unknowns even when dealing with nonlinear properties in the original problem. Numerical experiments were performed to demonstrate the optimal convergence of the scheme in the L^2 norm. These experiments involved specific choices of numerical fluxes combined with specific choices of approximation spaces.

Less is different: coherent structures make turbulence in one dimension difficult

Benno Rumpf
Southern Methodist University
brumpf@smu.edu

The theory of wave turbulence [1] provides an analytic connection between dynamical equations and statistical properties of a turbulent flow. It relies on the possibility of deriving closed kinetic equations for the slowly time dependent ensemble averages of the power; Kolmogorov-Zakharov spectra correspond to stationary non-equilibrium states of the kinetic equation. This picture changes when the formation of coherent structures provides an alternative mechanism of transferring power from sources to sinks in wavenumber space. The Majda-McLaughlin-Tabak equation [2-6] (which is a fractional nonlinear Schrödinger equation) is an example for this: previous work has shown that the formation and evolution of radiating pulses yields to a deformed power spectrum [3-5]. In my talk I will present my recent work on this system.

[1] A.C. Newell, B. Rumpf, Annu. Rev. Fluid Mech. 43, 59 (2011)
[2] A.J. Majda, D.W. McLaughlin, and E.G. Tabak, J. Nonlinear Sci. 7,9 (1997)
[3] V. Zakharov, F. Dias, and A. Pushkarev, Physics Reports 398, 1 (2004)
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[6] B. Rumpf, A.C. Newell, Phys. Lett. A 377, 1260 (2013)

SESSION 5

Peakons, kinkons, and other weak solutions for nonlinear wave equations

Invariant solutons of nonlnear mathematcal modelng of natural phenomena

Ranis N. Ibragimov
De Gruyter
ranis.ibragimov@degruyter.com

The main objective is to demonstrate the advantages of the invariance method in obtaining new exact analytic solutions expressed in terms of elementary functions for various physical phenomena. One application of the invariance method will be the mathematical modeling of oceanic and atmospheric whirlpools causing weather instabilities and, possibly, linked with climate change. As another particular example, it will be demonstrated that the invariance method allows to obtain the exact solutions of fully nonlinear Navier-Stokes equations within a thin rotating atmospheric shell that serves as a simple mathematical description of atmospheric circulation caused by the temperature difference between the equator and the poles with included equatorial flows modeling heat waves, known as Kelvin Waves. Special attention will be given to analyzing and visualizing the conserved densities associated with obtained exact solutions. As another modeling scenario, the exact solution of the shallow water equations simulating equatorial atmospheric waves of planetary scales will be analyzed and visualized. The presentation focuses mainly on the physical meaning and visualization the invariant solution rather than on mathematical derivations.

Peaked solitons and beyond

Zhijun (George) Qiao
University of Texas Rio Grande Valley
zhijun.qiao@utrgv.edu

In this talk, we will introduce integrable shallow water wave models given in scalar form, which possess peaked solitons (peakons), including the well-known Camassa-Holm (CH), the Degasperis-Procesi (DP), and other new peakon equations developed in recent years. In particular, the CH peakon equation is able to be extended to the DP, the b-family, the FORQ, the Novikov, the modified CH (MoCH), and other higher order models with peakons or pseudo-peakons. Open problems will also be addressed for discussion in the end. Some work is joint with UTRGV former students Miguel Rodriguez and Zhenteng Zeng, Prof. Baoqiang Xia, and Prof. Enrique Reyes.

Polynomial evolution equations over the octonions with Lax pairs

Thomas Wolf and **Stephen Anco**
Brock University
twolf@brocku.ca

Over recent decades many generalizations of integrable evolution equations have been studied, like vector versions, vector + scalar systems, matrix versions, complex and quaternion versions and one paper on the octonion version of KdV only. The talk will report on a *systematic* approach to find evolutionary equations with Lax pairs over the octonions.

The standard ansatz of homogeneous octonion polynomials with undetermined coefficients for the evolutionary equation $u_t = F$ and the Lax pair L, M needed to be adapted to non-associativity. Complications arose due to the unexpectedly large number of 10s of 1000s of octonion vanishing identities. A complete list of identically vanishing polynomials of degree up to 5 had been determined and used in the search.

Among the investigated homogeneity weights are $w(u)/w(\partial_x)$=2 (like KdV), =1 (like mKdV), =1/2 (like Ibragimov-Shabat) which provided octonion versions of KdV, mKdV, potential KdV and their higher order symmetries. Most of the F found are fully palindromic (symmetric when reversing factors but preserving brackets in each term) or fully antipalindromic but can also combine fully palindromic and fully anti-palindromic parts.

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Power series method for nonlinear fluid dynamics equations

Vesselin Vatchev
University of Texas Rio Grande Valley
vesselin.vatchev@utrgv.edu

In the talk we discuss a Frobenius method for studying partial differential equations with a polynomial type of nonlinearity. Equations of this type are regularly used to model the dynamics of fluids and are often related to the Euler equations. We show that we can obtain a wide range of traveling wave solutions to well-known equations, KdV, Boussinesq, Camassa-Holm, and more, by using the proposed method. For the classical Euler equations (EE) with a slightly modified kinematic equation we obtain an exact solution for the boundary conditions and the solution is a uniform approximation to the exact solution of the complete system. Furthermore, we show that in the context of asymptotic approximations the proposed method provides approximate solutions to the EE of exponential order. We also discuss properties, convergence, and extensions of the method.

Uniqueness of inverse spectral problem of non-local Sturm-Liouville operators on star graph

Lung-Hui Chen
Ming Chi University of Technology, Taiwan

In this paper, we explore the inverse spectral problem of Sturm-Liouville operator on a star-like graph. To this fixed star-like graph centered at the origin as its vertex, we attach m edges. On each edge, we impose the Sturm-Liouville operator with certain non-local potential functions with some suitable non-local boundary value conditions. At the vertex, we consider a frozen argument type of condition at zero to model a network that fixed on the end of each edge on the graph. The vibration and flow changes are monitored at that vertex which serves as certain control center. There is an inverse uniqueness subject to the suitable non-local boundary condition. We show that the system is solvable. Additionally, we give a Weyl's type of spectral asymptotics.

Peakons on a kink background

Stephen Anco
Brock University, Canada

Peakons are peaked travelling waves that were first found as solutions to the Camassa-Holm equation arising in the theory of shallow water waves. The Camassa-Holm equation is an integrable system which possesses multi-peakon solutions. These discoveries started an extensive study of integrable peakon equations.

A relatively unexplored topic is the study of peakons with nonzero boundary conditions at infinity. Some recent examples have been found in Refs. [1, 2] and describe a dynamical peakon (whose amplitude and speed are time dependent) propagating on a kink background which gets deformed by the motion of peakon and thereby becomes dynamical itself.

This talk will present some recent work investigating a simpler dynamical situation in which a dynamical peakon propagates on a static kink background, namely, the background remains undeformed during the motion of the peakon. These novel peakons are solutions of nonlinear dispersive convective equations $m_t + \alpha u_x + f(u,u_x)m + (g(u,u_x)m)_x = 0$, with $\alpha \neq 0$, where the nonlinearities f and g are quadratic. A variety of interesting dynamical behaviours are exhibited by the peakons, such as a finite-time extinction as the position goes to infinity, and an asymptotic slow down over a finite change in position.

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SESSION 6

Recent developments in nonlinear waves: From rogue waves and blow-ups to shocks, vortices and beyond

The phase space of the three-vortex problem

Atul Anurag and **Roy Goodman**
New Jersey Institute of Technology
aa2894@njit.edu, goodman@njit.edu

The three-vortex problem was first integrated by Gröbli in 1877. His method, which has been rediscovered and reused in many papers, introduces non-physical singularities. Other groups have introduced other changes of variables, each of which has suffered similar problems. We combine Jacobi coordinates with a recent Lie-Poisson reduction due to Ohsawa to obtain a near-universal reduction to the problem. We apply it to enumerate the system's global bifurcations as the vortices' circulations vary, and as the basis to study more complicated phenomena that emerge when a fourth vortex is added.

Spectral analysis of blow-up in nonlinear dispersive equations: Theory and computations

Efstathios G. Charalampidis
San Diego State University
echaralampidis@sdsu.edu

The spectral stability analysis of coherent structures in nonlinear dispersive PDEs is an important step towards understanding complex spatio-temporal phenomena. When a solution to a certain PDE is unstable, it is quite often that the instability leads to blow-up. In this talk, we will focus on the spectral stability analysis of blow-up to two prototypical 1D models, namely the (1+1)-dimensional generalized Nonlinear Schrödinger and Korteweg-de Vries (gNLS and gKdV, respectively) equations with general nonlinearity power by treating them as a bifurcation problem. Upon performing a dynamic rescaling on the models, we will present a general method that is capable of identifying self-similar waveforms as steady-state solutions in the so-called "co-exploding frame". Those emanate as bifurcating branches from the solitary branch at critical nonlinearity parameter. Then, we will bring forth bifurcation analysis techniques as well as computational methods associated with them in order to perform a spectral stability analysis of the pertinent waveforms over the nonlinear parameter. Conclusions will be drawn about how the instabilities identified in the co-exploding frame correspond to neutral, i.e., symmetry modes in the original frame, and thus are expected to be stable therein. If time permits, recent advances in multi-bump waveforms to the 1D gNLS will be discussed together with preliminary results in the 2D setup.

1. S. Jon Chapman, M. Kavousanakis, E.G. Charalampidis, I.G. Kevrekidis and P.G. Kevrekidis, A Spectral Analysis of the Nonlinear Schrödinger Equation in the Co-Exploding Frame, *Physica D* **439** (2022) 133396.
2. S. Jon Chapman, M. Kavousanakis, E.G. Charalampidis, I.G. Kevrekidis and P.G. Kevrekidis, Self-similar blowup solutions in the generalized Korteweg-de Vries equation: Spectral analysis, normal form and asymptotics, *Nonlinearity* **37** (2024) 095034.

Rogue energy fluctuations as extreme events may be the hallmark of non-integrable nonlinear systems

Rahul Kashyap[1], **Satya N. Majumdar**[2] and **Surajit Sen**[1]
[1]: State University of New York at Buffalo, [2]: Université de Paris-Sud, France
rahulkas@buffalo.edu, satyanarayan.majumdar@cnrs.fr, sen@buffalo.edu

Spatio-temporally extended kinetic energy fluctuations that exceed standard fluctuations by 6-8 fold or more are defined to be rogue energy fluctuations (REFs) in our studies. Earlier work based on dynamical simulations of non-integrable systems such as the energy conserved Hertz chains at zero loading (hence in the intrinsically nonlinear regime) held between fixed end walls and recent work on the Fermi-Pasta-Ulam-Tsingou-like system at purely nonlinear regimes reveal the emergence and sustained presence of significant REFs. The scattering processes during solitary wave collisions at sufficiently late times turn out to be the source of these REFs. With linear forces present along with nonlinear forces, the magnitudes of the REFs are less, as one might expect, but still present. They are, however, absent in linear systems. An examination of the statistics of the largest recorded REFs reveal a Gumbel distribution which suggests that the REFs follow extreme statistics of random events. Preliminary studies suggest that these phenomena are robust enough to be seen in purely quantum systems such as in $s = 1/2$ quantum spin chains.

Two-dimensional dispersive shock waves in the Kadomtsev-Petviashvili equation

Gino Biondini and **Alexander Bivolcic**
State University of New York at Buffalo
biondini@buffalo.edu
Mark Hoefer
University of Colorado, Boulder
hoefer@colorado.edu

I will discuss the oblique collisions and dynamical interference patterns of two-dimensional dispersive shock waves, which are studied numerically and analytically via the temporal dynamics induced by wedge-shaped initial conditions for the Kadomtsev-Petviashvili II equation. Various asymptotic wave patterns are identified, classified and characterized in terms of the incidence angle and the amplitude of the initial step, which can give rise to either subcritical or supercritical configurations, including the generalization to dispersive shock waves of the Mach reflection and expansion of viscous shocks and line solitons. An eightfold amplification of the amplitude of an obliquely incident flow upon a wall at the critical angle is demonstrated.

1. G. Biondini, A. Bivolcic and M. A. Hoefer, "Mach reflection and expansion of two-dimensional dispersive shock waves", arXiv:2411.05707 [nlin.ps]

Shock waves in the extended Gross-Pitaevskii equation

S. Chandramouli[1], **S.I. Mistakidis**[2], **G.C. Katsimiga**[2], **P.G. Kevrekidis**[1]
[1] University of Massachusetts
[2] Missouri University of Science and Technology
sathyanaraya@umass.edu, kevrekid@umass.edu, smystakidis@mst.edu,gkatsimiga@mst.edu

We demonstrate the controllable generation of distinct types of dispersive shock waves emerging in a quantum droplet bearing environment. Dispersive regularization of the ensuing hydrodynamic singularities occurs due to the competition between mean-field repulsion and attractive quantum fluctuations. The classification and characterization of these features are achieved by deploying Whitham modulation theory [1]. Our results pave the way for unveiling a multitude of unexplored coherently propagating waveforms in such attractively interacting mixtures and should be detectable by current experiments. Time permitting, we will discuss shock wave generation across multiple spatial dimensions.

1. Chandramouli et.al, Dispersive shock waves in a one-dimensional droplet-bearing environment, *Phys. Rev. A.*, **110** (2024), 023304 1-14.

Rogue curves in the Davey-Stewartson I equation

Jianke Yang
University of Vermont
jxyang@uvm.edu

We report new rogue wave patterns whose wave crests form closed or open curves in the spatial plane, which we call rogue curves, in the Davey-Stewartson I equation. These rogue curves come in various striking shapes, such as rings, double rings, and many others. They emerge from a uniform background (possibly with a few lumps on it), reach high amplitude in such striking shapes, and then disappear into the same background again. We reveal that these rogue curves would arise when an internal parameter in bilinear expressions of the rogue waves is real and large. Analytically, we show that these rogue curves are predicted by root curves of certain types of double-real-variable polynomials. We compare analytical predictions of rogue curves to true solutions and demonstrate good agreement between them. This is joint work with Dr. Bo Yang of Ningbo University.

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A novel geometric realization of the Yajima-Oikawa equations

Annalisa Calini and **Thomas Ivey**
College of Charleston
calinia@cofc.edu, iveyt@cofc.edu

In this talk we will discuss how the Yajima-Oikawa (YO) equations, a model of short wave-long wave interaction, arise from a simple geometric flow on curves in the 3-dimensional sphere that are transverse to the standard contact structure. For the family of periodic plane wave solutions of the YO equations studied by Wright, we construct the associated transverse curves, derive their closure condition, and exhibit several examples with non-trivial knot type.

1. A. Calini, T. Ivey, A Novel Geometric Realization of the Yajima-Oikawa Equations. *arXiv:2412.12946 preprint*, (2024), 13pp

Turbulence in driven dipolar gases across the superfluid-to-supersolid phase transition

Simeon I. Mistakidis and **George Bougas**
Missouri University of Science and Technology
smystakidis@mst.edu

We will discuss the nonequilbrium turbulent response of periodically driven magnetic gases by dynamically crossing the supersolid-to-superfluid phase transition and vice versa [1]. It is explicated that following defect formation (vortex lines), a direct energy cascade front emerges leading to a nonequilibrium quasi-steady state at longer evolution times. This quasi-steady state is characterized by the self-similar character of the momentum distributions featuring an algebraic reduction at large momenta with scaling exponents supporting wave turbulence. We shall further argue that supersolidity can be leveraged to promote turbulence. Moreover, different experimentally relevant protocols to excite inverse energy cascades with the aid of spatially dependent external potentials are elucidated. Finally, implications of the emergent dynamics to probe the solidity of the supersolid configurations will be showcased

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Collapse dynamics for two-dimensional space-time nonlocal nonlinear Schrödinger equations

Ziad Musslimani
Florida State University

The question of collapse (blow-up) in finite time is investigated for the two-dimensional (non-integrable) space-time nonlocal nonlinear Schr¨odinger equations. Starting from the two-dimensional extension of the well known AKNS q, r system, three different cases are considered: (i) partial and full parity-time (PT) symmetric, (ii) reverse-time (RT) symmetric, and (iii) general q, r system. Through extensive numerical experiments, it is shown that collapse of Gaussian initial conditions depends on the value of its quasi-power. The collapse dynamics (or lack thereof) strongly depends on whether the nonlocality is in space or time. A so-called quasi-variance identity is derived and its relationship to blow-up is discussed. Numerical simulations reveal that this quantity reaching zero in finite time does not (in general) guarantee collapse.

SESSION 7

Recent and alternate methods for the numerical solution of partial differential equations

Spectral representations in thermography

Jacob Z. Shapiro
BlueHalo
jacob.shapiro@bluehalo.com

Amanda K. Criner
Wright Patterson AFB
amanda.criner.1@afrl.af.mil

The basic experiment in thermal nondestructive evaluation is to heat the boundary of an object for a short duration with a flash lamp; the temperature within the object evolves according to a Robin boundary value problem for the heat equation. Measurements on the boundary are generally taken with an infrared camera, which records the average temperature over a given pixel.

Using this data, and representing the solution via a truncated eigenbasis for the Laplacian, we apply least squares or another standard optimization routine to estimate the fundamental eigenfrequencies of the domain. By the well established connection between the spectrum of a domain and its geometry (e.g., the classical Weyl and heat trace asymptotics), we use the eigenvalues to estimate the shape and size of interior defects like cavities or cracks.

Nondestructive evaluation techniques play an important role in safely using structures as they age beyond their design lives, including bridges, pipes, and aircraft. Spectral representations have played an important role since the early days of the subject.

We validate this approach for model domains in the plane with Neumann boundary condition. Finite element analysis yields the first ten to twenty eigenvalues, which are used in combination with the Weyl asymptotic and heat trace to find best fits to area and perimeter. We discuss uncertainty quantification and future approaches.

A novel multiscale sampling algorithm for subsurface characterization

Arunasalam Rahunanthan
Central State University
aRahunanthan@centralstate.edu
Abdullah Al Mamun and Felipe Pereira
The University of Texas at Dallas

Alsadig Ali
Hampton University

Abdullah Al-Mamun
United International University Dhaka, Bangladesh

Characterizing subsurface formations is challenging due to the high-dimensional stochastic space involved in solving inverse problems. To make this task computationally feasible, we apply the Karhunen-Loève Expansion (KLE) for dimensional reduction. Given the significant variation in rock properties such as permeability and porosity, it is beneficial to localize the sampling method. In a Bayesian framework, we address an inverse problem involving an elliptic partial differential equation for porous media flows. We introduce a novel multiscale sampling algorithm where the prior distribution is represented through local KL expansions in non-overlapping subdomains. The simulation results from a multiple Markov Chain Monte Carlo (MCMC) simulation study demonstrate that the proposed algorithm significantly enhances the convergence of a preconditioned MCMC method.

SESSION 8

Geometric methods in spectral theory of traveling waves

Faceting and folding in sharp interface gradient flows

Keith Promislow[1] and **Brian Wetton**[2]
[1]: Michigan State University, [2]: University of British Columbia, Canada
kpromisl@math.msu.edu

We give a systematic approach to the derivation of gradient flows from ‘intrinsic” free energies that characterize the state of a sharp interface solely in terms of its local structure. The Canham-Helfrich free energy is the classic example of an intrinsic sharp interface free energy. We apply the approach to a model energy for brine inclusion in ice in the presence of an antifreeze protein. The effect is an energetic preference for an interface with either a zero or high curvature interface – leading to a faceted interface. The facet-forming normal velocity inherits a Cahn-Hilliard type structure embedded within the evolution for the second fundamental form of the interface. We construct stationary solutions, and discuss their linear stability.

Adhesion-avoidance free energies are not intrinsic, they contain a convolution kernel that models energy of close self-approach of the interface. This requires information on relative cartesian distances of points on the interface. Generated by electrostatic effects, the kernel is repulsive at small distances and attractive at larger ones, like a Lennard-Jones potential. The avoidance component prevents interface self-intersection, balanced against the attractive longer range component this lead to ’folded’ equilibrium common in biological membranes. The fold structure balances adhesion energy against intrinsic bending energies. We construct global minima, gradient flows, and the structure of the linearization about equilibrium.

On the stability of smooth solutions to peakon equations

Stéphane Lafortune
College of Charleston
LafortuneS@cofc.edu
Brett Ehrman and **Mathew A. Johnson**
University of Kansas
ehrman.brett@ku.edu, matjohn@ku.edu
Dmitry Pelinovsky
McMaster University, Canada
dmpeli@math.mcmaster.ca

The Camassa-Holm equation with linear dispersion was originally derived as an asymptotic equation in shallow water wave theory. Among its many interesting mathematical properties, perhaps the most striking is the fact that it admits weak multi-soliton solutions - ‘peakons’- with a peaked shape corresponding to a discontinuous first derivative. Since the discovery of the Camassa-Holm equation, several peakon equations with similar properties, both in the integrable and non-integrable cases have been studied. Among them, there is the integrable Novikov equation, which can be regarded as a generalization to a cubic nonlinearity of the Camassa-Holm equation. Furthermore, both the Camassa-Holm and the Novikov equations each admit a generalization taking the form of a one-parameter family of peakon equations, most of which are not integrable. In this talk, we study the spectral and orbital stability of various smooth solutions to peakon equations. One of the main difficulties when dealing with the linear operators arising from peakon equation is that they often include a nonlocal term. An additional challenge is the fact that the localized smooth solutions admit a nonzero background.

Asymptotic stability of smooth solitons for the Camassa-Holm

equation

R.M. Chen
University of Pittsburgh
mingchen@pitt.edu
Y. Lan
Tsinghua University, P. R. China
lanyang@mail.tsinghua.edu.cn
Y. Liu
University of Texas at Arlington
yliu@uta.edu
Z. Wang
Foshan University, 528000, China
yliu@uta.edu

In this talk, we shall briefly report a recent asymptotic stability result of smooth solitons (and multi-solitons) in energy space for the Camassa-Holm (CH) equation. We show that a CH solution initially close to a soliton, once translated, converges weakly in H^1 to a possibly different soliton as time approaches to infinity. The proof is motivated by the bi-Hamiltonian structure of the CH equation and a Liouville type theorem for the CH flow close to the solitons. The new ingredient in the proof of Liouville theorem is by employing the completeness relations of square eigenfunctions of the CH recursion operator. Some applications are presented in classifications of solutions of linear problems related to KdV and mKdV equations. This is a joint work with Y. Lan, Y. Liu and Z. Wang.

Representations of strongly continuous cosine operator families in terms of the eigenvalues and the resonances of the generator

Alin Pogan
Miami University
pogana@miamioh.edu
Yuri Latushkin
University of Missouri
latushkiny@missouri.edu

We study representations of a strongly continuous cosine operator family, and the associated sine family, using contour integrals and eigenvalue and resonance expansions. This type of results are important to study the decay and oscillations of scattering waves or, more generally, of solutions of second order Cauchy problems in Banach spaces.

Transverse instability of Stokes waves

Ryan P. Creedon and **Walter A. Strauss**
Brown University
ryan_creedon@brown.edu, walter_strauss@brown.edu
Huy Q. Nguyen
University of Maryland
hnguye90@umd.edu

In 1847, George Stokes derived asymptotic expansions for the small-amplitude, periodic traveling-wave solutions of the full water wave equations. In the years since his work, much has been studied about these waves, including their stability (or lack thereof). Among the foundational results concerning the stability of Stokes waves was discovered by McLean in 1981 [1], who showed via numerical methods that these waves are unstable with respect to transverse perturbations of the initial data. A rigorous proof of this fact has been missing ever since. In this talk, I present the first ever proof of the existence of transverse instabilities of Stokes waves in both finite and infinite depth. In particular, it will be shown that the linearization of the full water wave equations about the Stokes waves admit solutions that grow exponentially in time.

1. McLean, J. W. and Ma, Y. C. and Martin, D. U. and Saffman, P. G. and Yuen, H. C., Three-Dimensional Instability of Finite-Amplitude Water Waves, *Phys. Rev. Lett.*, **46** (1981), 817-820.

Spectral stability via the Fredholm determinant of a trace class Birman-Schwinger operator

John Zweck and **Erika Gallo**
The University of Texas at Dallas
zweck@utdallas.edu, Erika.Gallo@utdallas.edu
Yuri Latushkin
University of Missouri
latushkiny@missouri.edu

We propose a novel computational method to determine the spectral stability of stationary pulse solutions of nonlinear wave equations such as the cubic-quintic complex Ginzburg-Landau equation in one spatial dimension. Specifically, we show that the point spectrum of the linearization of the equation about a pulse is given by the zero set of the regular Fredholm determinant of a trace-class Birman-Schwinger operator. This operator is defined in terms of a Green's kernel for the linearized equation. We adapt a method of Bornemann to numerically approximate the Fredholm determinant by a matrix determinant, and we quantify the error in this approximation. Numerical results show excellent agreement with existing methods. The new method avoids the computational challenge of solving a stiff system for the Jost solutions that is inherent in the standard approach based on computation of the Evans function. The motivation for developing the method was for future extensions to determine the spectral stability of time-periodic pulses, for which an Evans function most likely cannot be defined.

Stability for traveling fronts of the KdV-Burgers equation

Jared Bronski
University of Illinois
bronski@illinois.edu
Blake Barker
Brigham Young University
blake@math.byu.edu
Vera Hur
University of Illinois
verahur@illinois.edu
Zhao Yang
Chinese Academy of Sciences, China
yangzhao@amss.ac.cn

We investigate the stability of traveling front solutions to nonlinear diffusive-dispersive equations of Burgers type, with a primary focus on the Korteweg-de Vries–Burgers (KdVB) equation, although our analytical findings extend more broadly. Manipulating the temporal modulation of the translation parameter of the front and employing the energy method, we establish asymptotic, nonlinear, and orbital stability, provided that an auxiliary Schrödinger equation possesses precisely one bound state. Notably, our result is independent of the monotonicity of the profile and does not necessitate the initial condition to be close to the front. We identify a sufficient condition for stability based on a functional that characterizes the 'width' of the traveling wave profile. Analytical verification for the KdVB equation confirms that this sufficient condition holds for the relative dispersion parameter within an open interval $\supset [-0.25, 0.25]$, encompassing all monotone profiles. Utilizing validated numerics or rigorous computation, we present a computer-assisted proof demonstrating that the stability condition itself holds for parameter values within the interval $[.2533, 3.9]$.

Kinks of fractional ϕ^4 models: existence, uniqueness, monotonicity, and sharp asymptotics

Atanas Stefanov
University of Alabama - Birmingham
stefanov@uab.edu

We construct kink solutions for the fractional ϕ^4 model, in both the sub-Laplacian and super-Laplacian setting. We establish existence and monotonicity results (for the sub - Laplacian case), along with sharp asymptotics. Importantly, in the sub-Laplacian regime, we provide an explicit and numerically verify spectral condition, which guarantees uniqueness for odd kinks.

Floquet theory and stability analysis for Hamiltonian PDEs

J. C. Bronski and V. M. Hur
University of Illinois Urbana-Champaign

R. Marangell
University of Sydney, Australia

This talk is based on the recent work [1]. I will discuss some applications of Floquet theory to the stability and instability of periodic traveling waves in Hamiltonian PDEs. I will present several examples of such PDEs, including the generalized KdV and BBM equations (third order), the nonlinear Schrödinger and Boussinesq equations (fourth order), and the Kawahara equation (fifth order). The characteristic polynomial of the monodromy matrix inherits symmetry from the underlying PDE, and by employing the Floquet discriminant, one can determine properties of the essential spectrum along the imaginary axis including bifurcations of the spectrum.

1. Bronski, J. C., Hur, V. M., and Marangell, R. Floquet theory and stability analysis for Hamiltonian PDEs. *Nonlinearity*, **37** (12) (2024), 125010.

Existence and stability for the travelling waves of the Benjamin equation

Milena Stanislavova and **Atanas Stefanov**
University of Alabama Birmingham
mstanisl@uab.edu
Sevdzhan Hakkaev
Bulgarian Academy of Sciences
s.hakkaev@shu.bg

In the seminal work [3] in the late 70's, Benjamin derived the ubiquitous Benjamin model in the theory of water waves. It contains two parameters in its dispersion part and under some special circumstances, it turns into the KdV or the Benjamin-Ono equation. During the 90's, Benjamin, [4], [5] studied the problem for existence of solitary waves, followed by works of Bona-Chen, [6], Albert-Bona-Restrepo, [2], Pava, [7]. Some results about the stability became available, but they were restricted to either small waves or cases close to a distinguished (i.e. KdV or BO) limit. Quite recently, in [1], Abdallah, Darwich and Molinet, proved existence, orbital stability and uniqueness results for these waves, but only for large values of $\frac{c}{\gamma^2} >> 1$. In this talk, we present an alternative constrained maximization procedure for the construction of these waves in the full range of the parameters, which allows us to prove their spectral stability. Moreover, we extend this construction to all L^2 subcritical cases (i.e. power nonlinearities $(|u|^{p-2}u)_x$, $2 < p \leq 6$). Finally, we propose a different procedure, based on a specific form of the Sobolev embedding inequality, which works for all powers $2 < p < \infty$, but produces some unstable waves for large p.

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Orbital stability of smooth solitary waves in the Novikov equation

Mathew A. Johnson and **Brett Ehrman**
University of Kansas
matjohn@ku.edu, ehrman.brett@ku.edu
Stéphane Lafortune
College of Charleston
lafortunes@cofc.edu

We study the orbital stability of smooth solitary wave solutions of the Novikov equation, which is a Camassa-Holm type equation with cubic nonlinearities. These solitary waves are shown to exist as a one-parameter family (up to spatial translations) parameterized by their asymptotic endstate, and are encoded as critical points of a particular action functional. As an important step in our analysis we must study the spectrum the Hessian of this action functional, which turns out to be a nonlocal integro-differential operator acting on L^2. We provide a combination of analytical and numerical evidence that the necessary spectral hypotheses always holds for the Novikov equation. Together with a detailed study of the associated Vakhitov-Kolokolov condition, our analysis indicates that all smooth solitary wave solutions of the Novikov equation are nonlinearly orbitally stable.

Traveling front solutions in diffusive SIS model

Vahagn Manukian
Miami University
manukive@miamioh.edu
Anna Ghazaryan, Jonathan Waldmann, Priscilla Yinzime
Miami University
ghazarar@miamioh.edu, waldmaj3@miamioh.edu,
priscillayinzime1@gmail.com

We show the existence of traveling front solutions in diffusive SIS epidemic model in various parameter regimes. In the regime when the infected population diffuses slower than the susceptible population, we show the existence of traveling wave solutions for each fixed positive speed and describe their structure and dependence on the wave speed which is varied from 0 to infinity. In the regime when the infected population diffuses faster than the susceptible population, we show that the spread of the disease can be described by Burgers-FKPP equation and derive a bound for the speeds of the fronts in this regime.

Counting gap eigenvalues for the perturbed 3D nonlinear Schrödinger equation by a Maslov index

Emmanuel Fleurantin and **Christopher K.R.T. Jones**
George Mason University
efleuran@gmu.edu, cjone21@gmu.edu
Jeremy Marzuola
University of North Carolina at Chapel Hill
marzuola@math.unc.edu

We present a novel method for counting eigenvalues of the linearized JL operator associated with the perturbed 3D Nonlinear Schrödinger equation. Our study focuses on perturbations where the soliton and the potential operate at different scales. This separation is captured by rescaling the spatial variable, resulting in a fast-slow system. This approach allows us to isolate the effects of the potential from the intrinsic properties of the soliton and compare the perturbed system to its unperturbed counterpart. Our main result demonstrates that for sufficiently small perturbations, the number of eigenvalues of JL in the interval $(0, i) \subset i\mathbb{R}$ corresponds to the number of positive eigenvalues of the operator $\Delta - V$ on radial solutions. This counting is achieved through a generalization of Sturm-Liouville theory based on the Maslov index.

Spectral criteria for front invasion speeds

Montie Avery
Boston University
msavery@bu.edu

The emergence of complex spatial structures in physical systems often occurs after a simpler background state becomes unstable. Localized fluctuations then grow and spread into the unstable state, forming an invasion front which propagates with a fixed speed and mediates the creation of a new stable state in its wake. The *marginal stability conjecture*, recently proven for reaction-diffusion systems, asserts that the speed selected in the invasion process is the speed for which the associated front solution is marginally spectrally stable. In this talk, we explain how this conjecture gives a practical criterion for predicting invasion speeds, which can be efficiently verified numerically in large parameter regimes, or analytically near singular limits. We discuss applications to Lotka-Volterra systems, Keller-Segel models for bacterial motion, and a model for tumor growth.

Splitting quantum graphs

Alim Sukhtayev
Miami University
sukhtaa@miamioh.edu
Nathaniel Smith
Miami University
smithni@miamioh.edu

We derive a counting formula for the eigenvalues of Schrödinger operators with self-adjoint boundary conditions on quantum graphs. More specifically, using the Dirichlet-to-Neumann maps, which can be expressed as ratios of Evans functions, we develop techniques to reduce full quantum graph eigenvalue problems into smaller subgraph eigenvalue problems. These methods provide a simple way to calculate the spectra of operators with localized potentials.

Finite-time self-similar implosion of hollow vortices

Samuel Walsh
University of Missouri
walshsa@missouri.edu

In this talk, we will present some recent results on the finite-time blowup of hollow vortices. These are solutions of the two-dimensional Euler equations with the fluid domain being the complement of finitely many Jordan curves $\Gamma_1, \ldots, \Gamma_M$. The flow is irrotational and incompressible, but with a nonzero circulation around each boundary component. The "vortex core" bounded by each Γ_k is modeled as a bubble of ideal gas: the pressure is constant in space and inversely proportional to the area of the vortex. This can be thought of as the isobaric approximation.

The existence of collapsing configurations of point vortices is classical. We prove that generically, these can be desingularized to yield a families of hollow vortex configurations that exhibit self-similar finite-time implosion. Specific examples of an imploding trio and quartet of hollow vortices are given.

This is joint work with Ming Chen (University of Pittsburgh) and Miles Wheeler (University of Bath)

On the spectrum of the front in a predator-prey model

Anna Ghazaryan and **Vahagn Manukian**
Miami University
Ghazarar@miamioh.edu; Manukive@miamioh.edu
Stéphane Lafortune
College of Charleston
LafortuneS@cofc.edu
Yuri Latushkin
University of Missouri Columbia
Latushkiny@missouri.edu

We consider a predator-prey model with diffusion. Depending on the parameter regime, phenomenologically different types of fronts exist in this model. In particular, in the situation when the prey diffuses at the rate much smaller than that of the predator, there exists a parameter regime when the underlying dynamical system in a singular limit is reduced to a scalar Fisher-KPP equation [1]. The process of the reduction consists of taking limits with respect to two parameters. In this presentation, the stability of these fronts [2] is discussed. In particular, it is focused on obtaining uniform (in the singular parameters) bounds on the discrete spectrum.

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SESSION 9

Recent trends in acoustics and related wave phenomena: Computational and analytical methodologies

Krylov subspace spectral methods for problems in acoustics

James V. Lambers
The University of Southern Mississippi
James.Lambers@usm.edu

Krylov Subspace Spectral (KSS) methods are explicit time-stepping methods for PDEs in which each component of the solution, in some basis, is computed using an approximation of the solution operator of the PDE that is, in some sense, optimized for that component. This individualized approximation, based on techniques due to Golub and Meurant for approximating bilinear forms involving matrix functions, results in stability associated with implicit methods, as well as superior scalability as spatial resolution increases.

Within the last few years, it has been seen that KSS methods are particularly well-suited to various problems arising in acoustics. This talk provides an overview of recent and ongoing work in this area. This work includes (1) the use of techniques for approximating bilinear and quadratic forms involving matrix functions to measure the sensitivity of solutions of PDEs, (2) the application of KSS methods to the parabolic equation for acoustic pressure in the ocean, and (3) KSS methods for wave propagation problems featuring heterogeneous media and shock waves.

Numerical simulation of finite amplitude acoustic waves featuring gradient catastrophe

Chandler L. Shimp and **James V. Lambers**
The University of Southern Mississippi
Chandler.Shimp@usm.edu

This talk presents a hybrid analytical-numerical algorithm for the simulation of finite-amplitude acoustic waves, emphasizing moments of "gradient-catastrophe" or wave overturning. Our numerical method, that combines the method of characteristics with safeguarded root-finding methods and grid adaptivity, solves a 1-D hyperbolic nonlinear Cauchy problem that exhibits a physical shock and multi-valued solution profile. We demonstrate the efficacy of this root-finding method by computing solutions at each point in the space-time grid for different initial conditions with physical significance.

SESSION 10

Recent advances in nonlinear differential equations and applications

Nonlinear Liouville problem for the distribution of ice in the Arctic

Andrei Ludu
Embry-Riddle Aeronautical University
ludua@erau.edu

We develop a new physically data-driven model for the sea ice thickness distribution in the Arctic Ocean. We use a generic functional nonlinear approach to modify the existing evolution equation for the sea ice thickness distribution into a Liouville-like conservation law. The numerically generated solutions underlying our new equation governing the dynamics of the ice thickness predicts the observed sea-ice distribution shift function of the distribution of divergence and shear in the ice velocity field. Our approach provides a novel framework for studying the small and large scale structure of ice packs using methods of broad relevance in theory of distributions.

Inverse problems for tsunami waves

Oleksandr Bobrovnikov and **Alexei Rybkin**
University of Alaska Fairbanks
obobrovnikov@alaska.edu and arybkin@alaska.edu

We consider a long wave run-up in a U(V)-shaped inclined bay within the 1D non-linear shallow water equations (1D-SWE). This problem finds its applications in the tsunami modelling [2]. Typically, from the initial conditions (IC) of the wave, the initial displacement and velocity, one aims to compute the motion of the shoreline (wet/dry boundary). This problem is well-studied and can be solved using the classical Carrier-Greenspan hodograph (CGH) to linearise the 1D-SWE at a certain cost [1].

We study the following inverse problem: given the shoreline motion, find the IC. We first investigate the forward problem in order to separate the contribution of the velocity and the displacement to the shoreline motion, and then suggest a way to recover both IC.

We also show that the inverse problem in a way is better than the direct one. CGH complicates the IC in the hodograph space and some sophisticated tools are needed to overcome this hassle. The inverse problem does not suffer this complication and both IC are recovered exactly from the shoreline motion.

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G-Neutral functional equations with impulsive: existence, attracting and stability

Huoxia Liu and **Yongbing Luo**
Zhejiang A&F University, China
liuhx865578320@163.com; luoyongbingah@163.com

To understand the characteristics of dynamical behavior, especially in terms of attractiveness and stability. We investigate the effects of G-Brownian motion and impulses on the attractiveness and stability of solutions to neutral functional integro-differential equations. Based on resolvent operators theory, the Banach fixed point theorem and Borel-Cantelli lemma, the main ingredient of

this paper is to construct attracting and quasi-attracting sets, p-th moment stability and almost sure stability. Moreover, we attempt to study Hyers-Ulam and Ulam-Hyers-Rassias stability by a new technique, that is the Gronwell-type inequality. Finally, we give an example to verify our results.

Inclusions involving perturbed positively homogeneous maximal monotone operators

Dhruba R. Adhikari
Kennesaw University
dadhikar@kennesaw.edu

Let X be a real reflexive Banach space with X^* its dual. Let $L : X \supset D(L) \to X^*$ be a densely defined linear maximal monotone operator, $A : X \supset D(A) \to 2^{X^*}$ a maximal monotone and positively homogeneous operator of degree $\gamma > 0$, and $C : X \supset D(C) \to X^*$ a bounded demicontinuous operator of type (S_+) w.r.t. $D(L)$. Let G_1 and G_2 be open subsets X such that $\overline{G}_2 \subset G_1$, $0 \in G_2$, and G_1 is bounded. We discuss the existence of solutions of $Lx + Ax + Cx \ni 0$ in $G_1 \setminus G_2$ by reomving the restriction $\gamma = 1$ from the existing results. We also present applications to parabolic Dirichlet initial-boundary value problems.

SESSION 11

Water waves

Multiscale formulation of water waves: Derivation, modeling, and stability analysis

Katie Oliveras
Seattle University
oliveras@seattleu.edu

Abstract: In this talk, I will introduce a multiscale formulation for water wave dynamics that systematically extends the work of [1] to directly include multiple scales of motion. By redefining this foundational framework, we can easily derive various known model equations suitable for a range of boundary conditions, including decaying, periodic, generalized solitary waves, quasi-periodic, and various far-field background states. We will focus on two distinct model equations developed as approximations. Computational solutions for these models have been obtained, and their stability has been numerically analyzed. By comparing these model equations with the traditional Euler equations, we aim to better understand the scales at which specific phenomena occur. This comparison should help us develop more accurate reduced models that effectively capture essential features of the full water wave problem, such as modulational instabilities and high-frequency disturbances.

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High amplitude Stokes waves in a finite depth fluid

Eleanor Byrnes
University of Washington
elbyrnes@uw.edu

We apply a Newton-conjugate residual method to the conformal formulation of the one-dimensional Euler Water Wave Problem in terms of the horizontal spatial variable x and the vertical spatial variable y to find Stokes waves in a particular conformal depth. We find Stokes waves traveling in a fluid of a fixed physical depth through the use of a root-finding method. We compute the poles of these waves viewed as a function of the complex variable $z = x + iy$, identify any bifurcation points that occur as these waves increase in amplitude, and analyze the evolution of various physical quantities (energy, speed, steepness) as a function of amplitude through the lens of the infinite-depth asymptotic analysis done by Longuett-Higgins and Fox. Finally, we compute how the stability spectra of these waves evolve as we increase steepness. This is joint work with Bernard Deconinck, Anastassiya Semenova, and Sergey Dyachenko.

Homogenized equations for isentropic gas in a pipe with periodically-varying cross-section

Laila S. Busaleh and **David I. Ketcheson**
KAUST, Saudi Arabia
laila.busaleh@kaust.edu.sa, david.ketcheson@kaust.edu.sa

[Canceled]

In the solution of first-order nonlinear hyperbolic PDEs, shocks form generically if the coefficients are constant, whereas solitary waves may arise if the coefficients are periodically-varying. The periodic variation in the coefficients introduces microstructure-heterogeneity into the system. These heterogeneities can induce effective dispersion that prevents the formation of shocks, instead leading to the formation of solitary waves. In this talk, our focus lies in analyzing the behavior of a fluid propagating in a one-dimensional narrow pipe with periodically-varying cross-sectional area. The occurrence of these solitary waves is attributed to the coupling of nonlinearity and dispersive effects stemming from the medium's heterogeneity. We derive homogenized effective equations using multiple-scale perturbation theory which take the form of a constant-coefficient system including higher-order dispersive terms. We show that pseudospectral solutions to these equations agree well with direct solutions of the isentropic gas equation in periodically-varying cross-sectional area using a high-resolution finite-volume method.

Faraday waves and the spontaneous synchronization of oscillators

André Nachbin
Worcester Polytechnic Institute
anachbin@wpi.edu

Couder and collaborators [1] discovered that a droplet can bounce indefinitely and be self-propelled, on the surface of a vibrating bath. At each bounce, Faraday waves are triggered below the Faraday instability threshold, leading to a wave-particle system. This system, not imagined to exist in classical mechanics, has been shown to have many hydrodynamic quantum analogs [2].

In this presentation we will briefly describe the weakly viscous potential theory formulation for this system, in a domain with highly irregular cavities. The silicon-oil Faraday wave equations are very similar to water waves over non-smooth bottom topographies. Through the Schwarz-Christoffel mapping a Dirichlet-to-Neumann operator is formulated, allowing the 2D fluid-problem to be reduced to a 1D wave-particle system of equations. This coupled wave-droplet system is a novel nonlinear model for an active particle on a potential of its own making. When considering two bouncing droplets at a distance, trapped in their respective cavities, we observed a Kuramoto-like spontaneous synchronization of these two nonlinear oscillators [3], in regimes not before considered by the celebrated Kuramoto model. If time permits we will exhibit a case where two droplets/oscillators are correlated at a distance in the spirit of a static Bell test, violating the CHSH inequality [4].

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2. J.W.M. Bush and A.U. Oza, Hydrodynamic quantum analogs, *Rep. Prog. Phys.*, **84**, (2020) 017001.
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The spatial Whitham equation

John D. Carter
Seattle University
carterj1@seattleu.edu

Diane Henderson
Penn State University

Panos Panayotaros
Universidad Nacional Autónoma de México

The spatial Whitham equation is a recent generalization of the (temporal) Whitham equation that can be used to model the evolution of time-series measurements of the surface displacement of water waves. We compute periodic traveling-wave solutions to the spatial Whitham equation and examine their properties, including their stability. Second, we compare predictions from the spatial Whitham equation, its dissipative generalization, and classical nonlinear models with measurements from laboratory experiments.

1. J. D. Carter, D. M. Henderson, and P. Panayotaros, The spatial Whitham equation *Journal of Fluid Mechanics*, **996** (2024), A42.

Two-crested Stokes waves

Anastassiya Semenova
University of Washington
asemenov@uw.edu

The study of surface gravity waves is crucial for understanding the formation of rogue waves and whitecaps in ocean swell. Waves originating from the epicenter of a storm can often be approximated as unidirectional. In this presentation, we will explore periodic traveling waves on the free surface of an ideal, infinitely deep, two-dimensional fluid. We will focus on surface waves of permanent shape, and present new families of Stokes waves with two crests of different amplitudes per wavelength.

Modeling broad-frequency-band water-wave experiments

Joanna Van Liew and **John D. Carter**
Seattle University
jvanliew@seattleu.edu

Diane Henderson
Penn State

The nonlinear Schrödinger (NLS) equation and its generalizations model the evolution of modulated wave trains in deep water. We wrote Julia codes to solve the temporal NLS, dissipative NLS, Dysthe, viscous Dysthe, and dissipative Gramstad-Trulsen equations using operator splitting in time and a Fourier basis in space. We compared the predictions of these models with old and new experimental wave tank data. We found that the dissipative models significantly outperformed the conservative models for experiments involving narrow bands of frequencies. These narrow-band models also provided surprisingly accurate predictions for new broad-banded experiments. Our goal is to make comparisons with the new broad-band NLS generalization derived by Yan Li [1] which removes the narrow-bandwidth assumption.

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Collapsing breakers

Henrik Kalisch
University of Bergen, Norway
henrik.kalisch@uib.no

Breaking waves can be broadly classified in four categories: spilling, plunging, collapsing and surging [2]. For the simple case of a solitary wave running up on a slope, the type of breaker exhibited depends on the waveheight and the slope angle [3]. All types of wave breaking can be observed at various beaches, but collapsing breakers occur rarely and studies of collapsing breaker are few and far between.

We will look at some examples of wave breaking from field studies [1, 4], and then present a detailed study of a collapsing breaker in a wave laboratory [6]. In this case, the breaker dynamics are dominated by capillary effects, and the flow is similar to a capillary bore [5].

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4. H. Holand, H. Kalisch, M Bjørnestad, M Streßer, M.P. Buckley, J. Horstmann, V. Roeber, R. Carrasco-Alvarez, M. Cysewski and H.G. Frøysa, Identification of wave breaking from nearshore wave-by-wave records. *Phys. Fluids*, **35** (2023), 092105.
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6. W.Y. Wong, M. Bjørnestad, C. Lin, M.J. Kao, H. Kalisch, P. Guyenne, V. Roeber and J.M. Yuan, Internal flow properties in a capillary bore, *Phys. Fluids*, **31** (2019), 113602.

The Benjamin-Feir instability in KdV-like equations with general dispersion and monomial nonlinearity

Bernard Deconinck
University of Washington
deconinc@uw.edu

Nonlinear waves in dispersive media can exhibit modulation instabilities. We examine a category of scalar equations, with general dispersion and monomial nonlinearity, including a large variety of KdV-like equations. For small-amplitude traveling wave solution, we provide a complete characterization of the spectrum near the origin of the linear operator obtained from linearizing about periodic traveling waves. We prove rigorously that, when the modulational instability is present, the spectrum connected to the origin consists of curves that invariably form a closed figure-eight pattern. This is joint work with Bhavna Kaushik and Ashish Kumar Pandey.

Spatially quasi-periodic water waves

Xinyu Zhao
New Jersey Institute of Technology
xinyu.zhao@njit.edu
David Nicholls
University of Illinois at Chicago
davidn@uic.edu
Jon Wilkening
University of California, Berkeley
wilken@math.berkeley.edu

For linearized gravity-capillary waves, two periodic waves with different wavelengths may travel at the same speed. If the ratio of their wavelengths is irrational, the motion of their superposition becomes spatially quasi-periodic. We present a framework for computing and studying one-dimensional spatially quasi-periodic water waves, and we will also discuss an approach to extend this study to two-dimensional waves. This talk is based on joint work with David Nicholls and Jon Wilkening.

Instabilities of 2D periodic traveling water waves

Levent Batakci and **Bernard Deconinck**
University of Washington
Levent@uw.edu
David Nicholls
University of Illinois at Chicago

Using the framework due to Nicholls and Reitich [3], we compute 2-D traveling solutions to the classical water wave problem. The exact reformulation of the classical surface water wave problem due to Ablowitz, Fokas, and Musslimani[1] then provides a convenient framework to examine the spectral stability of these waves. We consider zero-average perturbations and linearize about the computed solutions, in line with the works of Deconinck and Oliveras on 1-D waves [4],[5]. The generalized Fourier-Floquet-Hill method [2] allows us to efficiently and accurately compute the stability spectra of these waves. We examine the instabilities of 2-D waves in both shallow and deep water, comparing our findings to the results for longitudinal and transverse perturbations of 1-D waves.

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The Stokes waves on ideal fluid: modulational instability and wave breaking

Sergey Dyachenko
State University of New York at Buffalo
sergeydy@buffalo.edu

The long-standing problem of linear stability of surface waves on 2D fluid is solved in conformal variables for Stokes up to nearly extreme steepness. The stability spectrum of Stokes waves exhibits recurrent transitions, multiple modulation, or Benjamin-Feir instability branches. We show that all Stokes waves are, in fact, unstable, but the nature of these instabilities varies – in some cases it leads directly to wave-breaking, and, in others, to modulational disturbance of ocean swell. We discuss the profound change in the numerical approach that allowed us to consider nearly extreme Stokes waves. New understanding of stability bifurcation is offered by series expansion of eigenvalue spectrum at the extrema of the Hamiltonian (and momentum).

SESSION 12

Singular asymptotics for integrable nonlinear waves and related topics

Random solitons, soliton gasses, and all that

Manuela Girotti
Emory University
manuela.girotti@emory.edu
Tamara Grava
SISSA and University of Bristol
grava@sissa.it
Ken McLaughlin
Tulane University
kmclaughlin@tulane.edu
Joseph Najnudel
University of Bristol
joseph.najnudel@bristol.ac.uk

The concept of a soliton gas was originally introduced by Zakharov in the Seventies and it can be loosely described as a class of solutions of nonlinear integrable PDEs that displays an infinite number of -possibly random- solitons.

I will present a collection of recent results from a collaborative effort to rigorously understand various aspects of several soliton gas models: long-time asymptotics, random statistics, and interaction dynamics. In particular, we consider a gas of random N solitons for the focusing nonlinear Schrödinger equation and we study the limit as N goes to infinity. We derive the limiting solution and we prove that its fluctuations are Gaussian random variables. The starting point is the formulation of a multi-soliton solution and of a regular, dense soliton gas solution in terms of Riemann–Hilbert problems. Tools from asymptotic analysis and probability will be used to provide a detailed description of interesting phenomena.

Long-time asymptotics for the KP I equation with small initial data

Samir Donmazov and **Peter Perry**
University of Kentucky
sdo244@uky.edu and peter.perry@uky.edu

Jiaqi Liu
University of Chinese Academy of Sciences, China
jqliu@ucas.ac.cn

The Kadomtsev-Petviashvili I (KP I) equation is a nonlinear dispersive partial differential equation modeling weakly dispersive waves in two dimensions. The inverse scattering transform (IST), formulated by Zhou [2] for the KP I equation, provides a framework for its analysis.

In our work [1], we rigorously obtain the long-time asymptotics of solutions for small initial data using Zhou's IST. The solution is decomposed into local and nonlocal components, both expressed as oscillatory integrals with cubic phase functions. The nonlocal component involves a time-dependent solution to a nonlocal Riemann-Hilbert problem (RHP).

We analyze asymptotics in three space-time regions: (1) non-degenerate stationary points, (2) a single degenerate stationary point, and (3) no stationary points. In all regions, we establish decay rates for both the local and nonlocal terms, uniform in the spatial variables x and y, with region-dependent rates: $O(t^{-1})$ in region (1), $O(t^{-2/3})$ in region (2), and $o(t^{-1})$ faster decay in region (3). As part of the asymptotic analysis for the nonlocal term, we determine the long-time asymptotics of the solution to the nonlocal Riemann-Hilbert problem and its x-derivative. To our knowledge, our estimates significantly improve previous results on this nonlocal Riemann-Hilbert problem.

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Borodin-Olshanski z-measures from the renormalization of quantum Benjamin-Ono periodic traveling waves

Alexander Moll
Reed College
amoll@reed.edu

In the reversible dynamics of the state $\Psi(t)$ of isolated quantum Hamiltonian systems, the spectral measure μ of the quantum Hamiltonian $\widehat{H}$ in the initial state $\Psi(0)$ plays a key role. If μ is absolutely continuous with respect to Lebesgue measure then the quantum survival probability $S(t) = |\langle\Psi(0), \Psi(t)\rangle|^2$ vanishes as $t \to \infty$. On the other hand, if μ is pure point then $S(t)$ is an almost periodic function of t and the system undergoes *quantum revivals*. Consider initial $\Psi(0) = \Upsilon_{u_0,\varepsilon}$ microlocalized around some classical initial data u_0 as $\varepsilon \to 0$. Here a dimensionless $0 < \varepsilon$ captures a separation of scales in which the initial expected energy is much greater than the ground state energy. For such initial data, a semiclassical description of the dynamics depends heavily on the nature of the underlying classical system.

In this paper, we consider the quantum revivals of initially coherent Gaussian wave packets $\Upsilon_{u_0,\varepsilon}$ in the quantum Benjamin-Ono equation on the torus. Using the exact description of the quantum spectrum of this system in terms of renormalized multi-phase solutions [1], the relevant spectral measure μ in this case is determined by a model of random Young diagrams known as *Jack measures*. For the Zabusky-Kruskal bell-shaped initial data $u_0(x) = 2\cos x$, these Jack measures become the well-known *Jack-Plancherel measures*. Our main result is that Borodin-Olshanksi's *z-measures* [2] arise naturally from the considerations above in the special case of initial data u_0 which define classical Benjamin-Ono periodic traveling waves. As a consequence, we derive new fluctuation results for z-measures in a new scaling limit by probing a semiclassical regime controlled by the linear stability of these traveling waves.

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Nonlinear steepest descent on a torus: A case study of the Landau-Lifshitz equation

Harini Desiraju
University of Sydney
harini.desiraju@sydney.edu.au

Alexander Its
Indiana University
aits@iu.edu

Andrei Prokhorov
University of Chicago
andreip@uchicago.edu

The Landau-Lifshitz equation describes the time evolution of spins in an anisotropic continuous spin chain (see [1]). It is an integrable nonlinear wave equation that follows the inverse scattering procedure established in [2]. We consider fast-decaying initial conditions without solitons and apply the Riemann-Hilbert method, set on a torus and developed in [3], for large-time asymptotic analysis.

First, we determine the class of scattering data that ensures the initial conditions belong to the Schwartz class. Then, using the solvability of the corresponding Riemann-Hilbert problem established in [4], we construct a singular integral equation representation for the solution, with a scalar Cauchy kernel defined on the torus. We proceed with the standard nonlinear steepest descent method, employing a parabolic cylinder parametrix. Finally, the asymptotics of the solution are obtained through the iterative analysis of a small-norm singular integral equation. Our result provides more rigorous derivation of formulas obtained in [5].

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Nonlinear steepest descent approach to the singular asymptotics of the radial Toda equation.

Maksim Kosmakov
University of Cincinnati
kosmakmm@ucmail.uc.edu

This work is a collaboration with M.Guest, A.Its, and K Miyahara, forming part of a broader project focused on studying the asymptotics of radial solutions to the 2D periodic Toda equation of type A_n.

In [GIK+23], it was shown that for $n = 2$ the monodromy manifold of the associated Lax pair for these equations consists of two connected components. The first component was demonstrated to correspond to solutions of the radial Toda equation. In this paper, we show that the second connected component of the monodromy manifold corresponds to solutions of a "partner" equation, which, after a change of variables, reduces to a special case of the Painlevé III (D_7) equation. Our main result is the description of the asymptotic behavior of these solutions as $x \to \infty$.

A significant technical challenge addressed in this work is the extension of the nonlinear steepest descent method to Riemann-Hilbert problems involving matrices of rank greater than two. Specifically, we analyze a 3×3 Riemann-Hilbert problem with singularities. While similar approaches have been successfully applied to 2×2 problems [FIS05, BI12], this work is the first to extend these techniques to higher-rank problems.

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Atypical focusing of semiclassical soliton ensembles in the AKNS hierarchy

Robert Buckingham
University of Cincinnati
buckinrt@ucmail.uc.edu

Robert Jenkins
University of Central Florida
robert.jenkins@ucf.edu

Peter Miller
University of Michigan
pdmiller@umich.edu

I will describe some recent work in which we investigate the small dispersion limit of the focusing NLS equation for solitonic approximations of parabolic pulse initial data. Talanov showed that solutions of the zero dispersion NLS equation with parabolic initial data exhibit singular self-focusing with finite-time blow up of their amplitude. Suleimanov recently proposed a mechanism for the dispersive regularization of this blow up. I'll discuss our proof of Suleimanov's regularization for NLS. We'll also discuss how this phenomena can be generalized to the higher order flows of the AKNS integrable hierarchy.

The semiclassical modified nonlinear Schrodinger equation with Talanov initial data

Chris DiScenza
University of Michigan
chrisdis@umich.edu

In the 1960's Talanov found a solution to the dispersionless nonlinear Schrodinger equation that blows up in finite time. Later, Suleimanov investigated the dispersive problem and found a solution that regularizes the blowup. We consider this problem for the modified nonlinear Schrodinger equation using the transformation of density and momentum variables found by DiFranco and Miller. However, the transformed Talanov solution mapped to the dispersionless modified nonlinear Schrodinger equation leads to an undefined lift-off in finite time before blowup. We attempt to find a solution that regularizes this lift-off phenomenon by solving the Riemann Hilbert problem and investigating the semiclassical limit.

Orthogonal polynomials and discrete Painlevé equations

Anton Dzhamay
Beijing Institute of Mathematical Sciences and Applications, China
adzham@bimsa.cn

We discuss different way discere Painlevé equations can appear in studying orthogonal polynomial ensembles. We also consider where such examples occur in the Sakai classification scheme for discrete Painlevé equations. We discuss connections between different ensembles and corresponding equations, both on the level of weight parameter limits and on the level of degenerations of the surface families.

On the asymptotics of Jacobi-type orthogonal polynomials on multiple intervals with non-analytic weights

Thomas Trogdon
University of Washington
trogdon@uw.edu

We consider the asymptotics of orthogonal polynomials for weight functions that are supported on multiple disjoint intervals. Weight functions are assumed to have Jacobi-type endpoint singularities but only a finite degree of smoothness. The Fokas-Its-Kitaev Riemann-Hilbert problem is analyzed with the dbar (Miller-McLaughlin) extension of the Deift-Zhou method of nonlinear steepest descent. We also discuss the addition of point masses and improvements for Chebyshev-type endpoint singularities.

Large-parameter asymptotics for rational solutions of Painlevé V

Trevor Johnson
University of Cincinnati
johns5tt@mail.uc.edu

We study the large-parameter asymptotics of rational solutions to Painlevé V using the method of nonlinear steepest descent. We derive a new Riemann-Hilbert representation for a family of rational solutions for Painlevé V built from generalized Laguerre polynomials. We present our new Riemann-Hilbert representation of Painlevé V rational solutions and discuss ongoing work to establish large-degree asymptotics for these rational solutions inside and outside the zero-pole region.

Asymptotics for orthogonal polynomials with applications in spectral density estimation

Charbel Abi Younes and **Tom Trogdon**
University of Washington
cyounes@uw.edu, trogdon@uw.edu

We examine the asymptotics of orthogonal polynomials related to both scalar and matrix eigenvector empirical spectral distributions (VESD). These asymptotics are derived by considering the discrete measures as perturbations of their continuous limits, where the asymptotics are more readily accessible. This asymptotic framework enables the estimation of both spikes and the VESD in $N \times N$ spiked sample covariance matrices as $N \to \infty$. We show that our approach provides accuracy up to an error of order $N^{-1/2}$ and are significantly faster than other available methods.

SESSION 13

Nonlinear partial differential equations and dynamical behavior of solutions

Global quantitative stability of wave equations with strong and weak dampings

Runzhang Xu
Harbin Engineering University, P R China
xurunzh@163.com; xurunzh@hrbeu.edu.cn

In this talk, we are concerned with the description of global quantitative stability of wave equations with linear strong damping and linear or nonlinear weak damping. By giving some energy decay estimates, we obtain several conclusions about the continuous dependence of the global solution on the initial data and the coefficients of the strong damping term and linear or nonlinear weak damping term. This work also establishes a new idea to use the dissipative effect to obtain the better continuous dependence conclusions, which also reflect the dissipative properties of the solution. This is a collaborated work with Dr. Jiangbo Han at Inner Mongolia University&Harbin Engineering University, Dr. Chao Yang at AGH University of Science and Technology&Harbin Engineering University, and Professor Keyan Wang at Shanghai University of Finance and Economics.

Nonlinear partial differential equations and dynamical behavior of solution

Wei Lian and **Erik Wahlén**
Lund University, Sweden
lianwei_1993@163.com, wei.lian@math.lu.se, erik.wahlen@math.lu.se

The passage from linear instability to nonlinear instability has been shown for 1D solitary waves under 2D perturbations. Although transverse instability of periodic waves to the KdV equation under the KP-I flow has been expected to be true from spectral instability for a long time, it has not been clear how to adapt the general instability theory for solitary waves to periodic waves until now. In this talk, we present how such an adaptation works with the aid of exponential trichotomies and multivariable Puiseux series.

Well-posedness for $p(x)$-Laplacian parabolic equations with multiple regime on an annulus

Yitian Wang
Harbin Engineering University, P R China
wangyitian@hrbeu.edu.cn

This study investigates the well-posedness of solutions to the initial boundary value problem for parabolic equations with variable exponents of multiple regime (subcritical, critical, and supercritical) on an annulus. The presence of critical and supercritical regimes disrupts classical Sobolev embeddings, leading to the lack of compactness. To address these issues, we use the Strauss inequality to restore compact Sobolev embeddings for radially symmetric functions. By employing the subdifferential technique with symmetry constraints, we establish local existence of solutions for any radially symmetric initial data and demonstrate uniqueness. We pioneer the application of the potential well theory to classify initial data based on three energy levels: subcritical, critical, and supercritical. For subcritical and critical levels, we analyze cases with non-positive and positive initial energy, obtaining results on finite-time blowup and identifying threshold conditions for global existence versus blowup. Finally, we extend these results to a broader class of locally symmetric domains containing an annulus. This is a collaborated work with Professor Runzhang Xu and Dr. Chao Yang at Harbin Engineering University.

Global bifurcation of three-dimensional gravity-capillary waves on Beltrami flows

Bastian Hilder
Technical University of Munich, Germany
bastian.hilder@tum.de
Erik Wahlén and **Giang To**
Lund University, Sweden
erik.wahlen@math.lu.se, giang.to@math.lu.se

Most of the current theory on three-dimensional steady water waves assumes irrotational flow. One exception is the construction of a family of small-amplitude doubly periodic gravity-capillary waves on Beltrami flows by Erik Wahlén, Lokharu and Svensson Seth from 2020. In my talk I will describe a global continuation of this family. One of the challenges is that the local family is constructed using a multiparameter bifurcation approach, whereas global bifurcation theory usually assumes a single bifurcation parameter. Our theory includes irrotational flow as special case.

Solitary axisymmetric capillary water waves

Stefano Böhmer
Lund University, Sweden
stefano.bohmer@math.lu.se

We consider steady axisymmetric water waves subject to surface tension, where we study the free-boundary problem for domains close to an infinite cylinder. In the case of linear vorticity in radial direction and no swirl, we are able to prove existence of small solitary solutions of KdV-type. They bifurcate from laminar flows in a flat cylinder and the presence of vorticity is required for their existence. The proof relies on a spatial dynamics approach allowing for a center-manifold reduction, which reduces the problem to a finite-dimensional dynamical system. Homoclinic solutions of this system, which correspond to solitary wave solutions on the cylinder, are found using dynamical systems methods.

This is joint work with Dag Nilsson (Mid Sweden University).

Global existence and finite time blowup for an anisotropic parabolic equation in weighted variable Sobolev spaces

Yue Pang
Heilongjiang University, P R China
pangyue57@163.com; pangyue@hlju.edu.cn

In this talk, we will present the global well-posedness of the solution for an anisotropic parabolic equation on the bounded domains with Dirichlet boundary conditions in weighted variable Sobolev spaces. We prove the local existence and uniqueness of the solution by the contraction mapping principle. We further give a sharp condition for the global solution and the finite time blowup solution at the sub-critical and the critical initial energy levels, respectively. We also make the decay estimates of the global solution and the upper bound of the blowup time at above two initial energy levels. The finite time blowup of the solution is proved at the sup-critical initial energy level. Finally, the lower bound of the blowup time is showed in some suitable condition for any initial energy level. This is a collaborated work with Professor Runzhang Xu at Harbin Engineering University.

SESSION 14

Recent advances in algorithmic development for nonlinear PDEs

Incremental data compression for PDE-constrained optimization

Xuejian Li
Clemson University

In this talk, we present an inexact gradient method based on incremental proper orthogonal decomposition (iPOD) to address the data storage difficulty in time-dependent PDE-constrained optimization. We will employ techniques from Hilbert-Schmidt operators, numerical PDEs, and convex optimization to prove that the proposed method is robust. More interesting, this inexact gradient method is manageable to achieve the accuracy level of the optimal solution while not hurting the convergence rate compared with the usual gradient method. We also use numerical experiments to verify the theoretical results and illustrate the proposed method.

Exponential time differencing method for a reaction-diffusion system with free boundary

Xinfeng Liu
University of South Carolina
xfliu@math.sc.edu

For reaction-diffusion equations in irregular domains with moving boundaries, the numerical stability constraints from the reaction and diffusion terms often require very restricted time step sizes, while complex geometries may lead to difficulties in accuracy when discretizing the high-order derivatives on grid points near the boundary. It is very challenging to design numerical methods that can efficiently and accurately handle both difficulties. Applying an implicit scheme may be able to remove the stability constraints on the time step, however, it usually requires solving a large global system of nonlinear equations for each time step, and the computational cost could be significant. Integration factor (IF) or exponential differencing time (ETD) methods are one of the popular methods for temporal partial differential equations (PDEs) among many other methods. In our paper, we couple ETD methods with an embedded boundary method to solve a system of reaction-diffusion equations with complex geometries. In particular, we rewrite all ETD schemes into a linear combination of specific ϕ-functions and apply one state-of-the-art algorithm to compute the matrix-vector multiplications, which offers significant computational advantages with adaptive Krylov subspaces. In addition, we extend this method by incorporating the level set method to solve the free boundary problem. The accuracy, stability, and efficiency of the developed method are demonstrated by numerical examples.

Parallel and energy-preserving schemes based on the partitioned averaged vector field method

Xuelong Gu
University of South Carolina
2513977802@qq.com

In this talk, we introduce a novel approach for developing highly efficient, energy-preserving methods for both conservative and dissipative partial differential equations (PDEs). Our method utilizes spatial finite difference approximations and, while it appears fully implicit, it is actually completely decoupled point by point. For the sine-Gordon and Allen-Cahn equations, our implementation only requires solving a scalar nonlinear equation successively, resulting in a complexity that scales with the degrees of freedom. Additionally, for the coupled Klein-Gordon-Schrodinger equations, the scheme enables fully explicit computation. Building on the partitioned averaged vector field method, we further generalize the proposed schemes to facilitate naturally parallel computation, significantly enhancing computational efficiency. Various numerical experiments are presented to demonstrate the performance of the proposed schemes.

General numerical framework for structure-preserving reduced order models of thermodynamically consistent reversible-irreversible PDEs

Zengyan Zhang and **Jia Zhao**
Pennsylvania State University, University of Alabama
jia.zhao@ua.edu

In this talk, I will present a newly developed numerical framework to derive structure-preserving reduced order models for thermodynamically consistent PDEs [1]. Our approach focuses on two key aspects: (a) a systematic method for generating reduced order models that respect the underlying thermodynamic principles of the original PDE systems, and (b) a strategy for constructing accurate, efficient, and structure-preserving numerical algorithms to solve these reduced order models. The framework's generality allows it to be applied to a wide range of PDE systems governed by thermodynamic laws. We will demonstrate the effectiveness of this approach through several numerical examples

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SESSION 15

Recent advances in stability of nonlinear waves

Linearized KdV equation on the line with a metric graph defect

Dave Smith
University of Newcastle, Australia
dave.smith@newcastle.edu.au

The small amplitude linearization of the KdV equation can be solved on the line via the Fourier transform, but on domains with a boundary induce more complex spectral properties, requiring more specialized transforms. We show how the unified transform method can be adapted to study this equation on metric graph domains with two semiinfinite leads and some assembly of bonds attached between them. We discuss how a perturbative analysis can lead to stability results for the nonlinear KdV equation in such a setting.

Well-posedness, stability, and bifurcation for Keller-Segel models on compact graphs

Hewan Shemtaga and **Wenxian Shen**
Auburn University
hms0069@auburn.edu, wenxish@auburn.edu
Selim Sukhtaiev
Auburn University
szs0266@auburn.edu

In this talk, we will discuss the Keller-Segel model describing various chemotaxis processes. The system of PDEs in question is a pair of reaction-advection-diffusion equations of parabolic and elliptic type. The first part of the talk will center around well-posedness of this system on arbitrary compact metric graphs. In the second part of the talk, we will focus on asymptotic stability, instability, and bifurcation of steady state solutions of parabolic-parabolic and parabolic-elliptic chemotaxis models. This is joint work with Hewan Shemtaga (Auburn) and Wenxian Shen (Auburn).

SESSION 16

Recent developments on free surface flows

Proof of the transverse instabilities of Stokes waves

Huy Nguyen
University of Maryland
hnguye90@umd.edu
Ryan Creedon
University of Washington
creedon@uw.edu
W. Strauss
Brown University
wstrauss@math.brown.edu

A Stokes wave is a traveling free-surface periodic water wave that is constant in the direction transverse to the direction of propagation. It was discovered in the 1980s via numerical methods that small-amplitude Stokes waves are unstable with respect to transverse perturbations of the initial data. We will discuss our recent rigorous proof of these transverse instabilities in both the infinite and finite depth cases.

On bounds for the Froude number for solitary water waves

Jörg Weber
University of Wien, Austria
joerg.weber@univie.ac.at

A classical and central problem in the theory of water waves is to classify parameter regimes for which nontrivial solitary waves can and do exist. In the two-dimensional, irrotational, pure gravity case, the non-dimensional Froude number $F = c/\sqrt{gd}$ (with c the wave speed, g the gravitational constant, and d the depth of the fluid at infinity) plays the central role. So far it is known that necessarily $F \in (1, \sqrt{2})$ for a nontrivial solitary wave. While the lower bound is sharp, numerics indicate that the upper bound is *not* sharp. In this talk, we discuss a new strategy utilising the flow force function and rigorously establish an improved upper bound for F. Moreover, we outline similar results in the case of constant vorticity. The talk is based on joint work with Evgeniy Lokharu (Lund).

Thermal effects on the deformation of a gas bubble in an incompressible liquid

Chen-Chih Lai
Columbia University
cclai.math@gmail.com
Michael I. Weinstein
Columbia University
miw2103@columbia.edu

We study the thermal decay of bubble oscillations in an incompressible fluid with surface tension. Particularly, we focus on the isobaric approximation [Prosperetti, JFM, 1991], under which the gas pressure within the bubble is spatially uniform and follows the ideal gas law. This model exhibits a one-parameter family of spherical equilibria, parametrized by the bubble mass. We prove that this family forms an attracting centre manifold for small spherically symmetric perturbations, with solutions converging to the manifold at an exponential rate over time. Furthermore, we show that under either liquid viscosity or irrotational flow assumptions, any equilibrium gas bubble must be spherical by proving that the bubble boundary is a closed surface of constant mean curvature. Additionally, the manifold of spherically symmetric equilibria captures all regular spherically symmetric equilibrium.

We also explore the dynamics of the bubble-fluid system subject to a small-amplitude, time-periodic, spherically symmetric external sound field. For this periodically forced system, we establish the existence of a unique time-periodic solution that is nonlinearly and exponentially asymptotically stable against small spherically symmetric perturbations.

In the latter part of the talk, I will discuss some limitations of the isobaric model in a more general (nonspherically symmetric) irrotational setting. Specifically, I will address issues such as (1) the undamped oscillations of shape modes due to spatial uniformity of the gas pressure, and (2) the incompatibility between viscosity and irroataionality assumptions. Our results suggest that to accurately capture the effect of thermal damping on the dynamics of general deformations of a gas bubble, the model should be considered within a framework that includes either non-zero vorticity, corrections to the isobaric approximation, or both.

If time permits, I will present ongoing work on the existence of nonspherically symmetric equilibrium bubbles in a rotational framework.

This talk is based on joint work with Michael I. Weinstein ([Arch. Ration. Mech. Anal. 2023], [Nonlinear Anal. 2024], [arXiv:2408.03787], and work in progress).

Vortex-carrying solitary gravity waves of large amplitude

Robin Ming Chen and **Kristoffer Varholm**
University of Pittsburgh
mingchen@pitt.edu, kristoffer.varholm@pitt.edu

Samuel Walsh
University of Missouri
walshsa@missouri.edu

Miles H. Wheeler
University of Bath, UK
mw2319@bath.ac.uk

In this paper, we study two-dimensional traveling waves in finite-depth water that are acted upon solely by gravity. We prove that, for any supercritical Froude number (non-dimensionalized wave speed), there exists a continuous one-parameter family C of solitary waves in equilibrium with a submerged point vortex. This family bifurcates from an irrotational uniform flow, and, at least for large Froude numbers, extends up to the development of a surface singularity or blowup of the circulation. These are the first rigorously constructed gravity wave-borne point vortices without surface tension, and notably our formulation allows the free surface to be overhanging. We also provide a numerical bifurcation study of traveling periodic gravity waves with submerged point vortices, which strongly suggests that some of these waves indeed overturn. Finally, we prove that at generic solutions on C — including those that are large amplitude or even overhanging — the point vortex can be desingularized to obtain solitary waves with a submerged hollow vortex. Physically, these can be thought of as traveling waves carrying spinning bubbles of air.

SESSION 17

Fractional calculus and its applications

H3N3-2_σ-based finite difference method for the fractional diffusion-wave equations

Changpin Li
Shanghai University, China
lcp@shu.edu.cn

A novel H3N3-2_σ approximate formula for the Caputo fractional derivative of order $\alpha \in (1,2)$ is established with the second-order accuracy, which improves the popular L2C formula with (3-α)-order accuracy. Based on this formula, a finite difference scheme with second-order accuracy both in time and in space is constructed for the initial-boundary value problem of the time fractional diffusion-wave equation [1]. Such a method with technique is generalised to the time multi-term fractional diffusion-wave equation [2]. The numerical simulations are performed to show the effectiveness of the derived finite difference schemes, in which the fast algorithms are also employed to speed up the numerical computations [1, 2].

1. R. L. Du, C. P. Li, Z. Z. Sun, H^1-analysis of H3N3-2_σ-based difference method for fractional hyperbolic equations, *Comput. Appl. Math.*, **43**(1) (2024), 69: 1-34.
2. R. L. Du, C. P. Li, F. Su, Z. Z. Sun, H3N3-2_σ-based difference schemes for time multi-term fractional diffusion-wave equation, *Comput. Appl. Math.*, **43**(8) (2024), 416: 1-35.

SCQ for fractional calculus

Yang Liu
Inner Mongolia University, China
mathliuyang@imu.edu.cn

In this talk, we discuss the developed (SCQ), which covers most difference formulas from the aspects of characterizing the formation of related generating functions. For stability reasons, the theoretical determination of shifted parameter θ is provided to fill the gap in which the choice of θ depends heavily on experiments particularly for non-integer order derivatives. We develope some numerical methods based on the SCQ framework to fractional PDEs, and discuss their numerical theories and computing results.

Global Mittag-Leffler stability of the delayed fractional-coupled reaction-diffusion system on networks without strong connectedness

Yue Cao
Inner Mongolia University, China
supercyue@163.com

In this paper, we mainly consider the existence of solutions and global Mittag-Leffler stability of delayed fractional-order coupled reaction-diffusion neural networks without strong connectedness. Using the Leary-Schauder's fixed point theorem and the Lyapunov method, some criteria for the existence of solutions and global Mittag-Leffler stability are given. Finally, the correctness of the theory is verified by a numerical example.

A high-order discrete energy decay and maximum-principle preserving scheme for time fractional Allen-Cahn equation

Guoyu Zhang
Inner Mongolia University, China
guoyu_zhang@imu.edu.cn

The shifted fractional trapezoidal rule (SFTR) with a special shift is adopted to construct a finite difference scheme for the time-fractional Allen-Cahn (tFAC) equation. Some essential key properties of the weights of SFTR are explored for the first time. Based on these properties, we rigorously demonstrate the discrete energy decay property and maximum-principle preservation for the scheme. Numerical investigations show that the scheme can resolve the intrinsic initial singularity of such nonlinear fractional equations as tFAC equation on uniform meshes without any correction. Comparison with the classic fractional BDF2 and L2-1_σ method further validates the superiority of SFTR in solving the tFAC equation. Experiments concerning both discrete energy decay and discrete maximum-principle also verify the correctness of the theoretical results.

SESSION 18

Existence and stability of traveling waves

Uniform bounds of families of analytic semigroups and Lyapunov linear stability of planar fronts

Yuri Latushkin and **Alin Pogan**
University of Missouri and Miami University
latushkiny@missouri.edu and pogana@miamioh.edu

We study families of analytic semigroups, acting in a Banach space, and depending on a parameter, and give sufficient conditions for existence of uniform with respect to the parameter norm bounds using spectral properties of the respective semigroup generators. In particular, we use estimates of the resolvent operators of the generators along vertical segments to estimate the growth/decay rate of the norm for the family of analytic semigroups. These results are applied to prove the Lyapunov linear stability of planar traveling waves of systems of reaction-diffusion equations, and the bidomain equation, important in electrophysiology.

Multi-modal solitary wave solutions to a fourth-order nonlinear Schrödinger equation

Ross Parker
Center for Communications Research - Princeton
r.parker@idaccr.org

In this talk, we consider the existence and spectral stability of multi-modal solitary wave solutions to a nonlinear Schrödinger equation incorporating both fourth and second-order dispersion terms. In the bright soliton regime, we show that we can construct multi-modal solitary waves by "gluing together" consecutive copies of the primary solitary wave, as long as certain geometric constraints are satisfied. Under additional assumptions, which can be verified numerically, we prove that all such multi-pulses are spectrally unstable. By contrast, numerical results suggest that, in the dark soliton regime, some multi-modal solutions are in fact stable.

Existence of asymmetric grain boundaries

Qiliang Wu
Ohio University
Pwuq@ohio.edu

Grain boundaries, both asymmetric and symmetric ones, commonly observed in nature, experiments and numerical simulations as one of the basic defects of spatially periodic patterns. The existence of asymmetric grain boundaries is established in the Swift-Hohenberg equation posed on the whole Euclidean plane. Similar to the previous results for the symmetric case, we exploit spatial dynamics and the center manifold reduction to reduce the existence problem of these solutions in the stationary Swift-Hohenberg equation—a fourth order elliptic PDE, to the existence of heteroclinic orbits in a reduced ODE system. However, the establishment of these heteroclitic orbits in the reduced ODE system is significantly different from the symmetric case and thus the main result of this work. More specifically, we first show the existence of heteroclinic orbits in the normal form up to the cubic orders via the sliding method. We then show the persistence of these heteroclinic orbits in the full reduced ODE system via an implicit function theorem argument. Along with the existence, it is shown that wavenumbers of roll patterns in the the two far fields away from the grain boundary, together with the far-field phase shift, is selected by the periodicity along the grain boundary and the two rotated angles of the roll patterns in the far field.

Kinks in nonlinear Klein-Gordon models: From variable nonlinearity to fractional dispersion

P.G. Kevrekidis
UMass, Amherst
kevrekid@umass.edu

In the present work, we will revisit Klein-Gordon models involving the nonlinearity of the prototypical, so-called, ϕ^4 model. We will consider various extensions of these models. Firstly, we will modify their nonlinearity from ϕ^4 to ϕ^6 and then to ϕ^8, and beyond. We will see how the solitary waves change and how they may acquire power-law tails (e.g., in the ϕ^8 model) and how their interactions become power-law in nature. We will then modify the system's dispersion by considering quartic models with the ϕ^4 nonlinear potential, as well as combinations of the quadratic and quartic derivatives, to observe how their potential competition may shape up the nature of the wave interactions. Finally, in the spirit of fractional dispersive models, we consider an example of a fractional Riesz (spatial) derivative interpolating between the case of integer orders 2 and 4 and see how the fractional model transitions between the harmonic and the biharmonic ϕ^4 model. As an aside, the effect of a Caputo temporal derivative will be considered as well.

Transverse spectral instability of the Novikov-KP equation

Brett Ehrman and **Mathew A. Johnson**
University of Kansas
ehrman.brett@ku.edu, matjohn@ku.edu

The Novikov equation is a nonlinear dispersive PDE that models the unidirectional propagation of shallow water waves, similar to the Camassa-Holm equation but with cubic nonlinearity. The Novikov equation has (up to translations) a three-parameter family of smooth, periodic traveling wave solutions. Knowing that there are such spectrally stable solutions (with respect to localized perturbations) of the Novikov equation in the asymptotically small-amplitude regime, we consider these as solutions to the Novikov-KP equation, which generalizes the Novikov equation to two dimensions. Thus, we study the spectral stability of these solutions, subject to two-dimensional perturbations that (in the direction of propagation) are localized or have the same period as our one-dimensional solution. The Novikov-KP equation has two cases, called the Novikov-KP-I and Novikov-KP-II equations, depending on whether the dispersion parameter is chosen to be positive or negative. We find that the sign of the dispersion parameter greatly affects our spectral stability/instability results.

Resonant bifurcations: Waves and eigenvalues

Benjamin Akers
Air Force Institute of Technology
benjamin.akers@us.af.mil

[Canceled]

Asymptotic and numerical computations of traveling waves and their spectrum in model equations for waters waves are presented. An analogy is made between bimodal traveling waves, e.g. Wilton Ripples, and the spectral data near eigenvalue collisions. An expansion that yields sheets of waves is discussed in the context of the spectral data.

SESSION 19

Stability for nonlinear waves with singularities

Stability of close-to-couette shear flows in a finite channel

Siming He
University of South Carolina
siming@mailbox.sc.edu

In this talk, I will present a threshold theorem for the 2D Navier-Stokes equations posed on the periodic channel, supplemented with Navier boundary conditions. The initial datum is taken to be a suitable perturbation of a shear flow that is close to the Couette flow. For such a datum, we prove nonlinear enhanced dissipation and inviscid damping for the resulting solution. The principal innovation is to capture quantitatively the inviscid damping, for which we introduce a new Singular Integral Operator (SIO). We combine the SIO with the hypocoercivity functional to derive the stability result. This is joint work with Jacob Bedrossian, Sameer Iyer, and Fei Wang.

Fast-slow dynamics of the quasi-geostrophic approximation

Claude Bardos
Denis Diderot University Paris, France
claude.bardos@gmail.com
Xin Liu
Texas A&M University, Texas
xliu23@tamu.edu
Edriss Titi
University of Cambridge, UK
Edriss.Titi@damtp.cam.ac.uk

This work is devoted to investigating the rotating Boussinesq equations of inviscid, incompressible flows with both fast Rossby waves and fast internal gravity waves. The main objective is to establish a rigorous derivation and justification of a new generalized quasi-geostrophic approximation in a channel domain with no normal flow at the upper and lower solid boundaries, taking into account the resonance terms due to the fast and slow waves interactions. Under these circumstances, We are able to obtain uniform estimates and compactness without the requirement of either well-prepared initial data or domain with no boundary. In particular, the nonlinear resonances and the new limit system, which takes into account the fast waves correction to the slow waves dynamics, are also identified without introducing Fourier series expansion. The key ingredient includes the introduction of (full) generalized potential vorticity.

1. Bardos, C., Liu, X. and Titi, E.S., 2024. Derivation of a generalized quasi-geostrophic approximation for inviscid flows in a channel domain: The fast waves correction. *Communications in Mathematical Physics*, **405(7)** (2024), p.164.

L^2 theory for compressible fluid

Geng Chen
University of Kansas

In this talk, I will introduce the recent advances in the L^2 theory for compressible Euler and Navier Stokes equations, including the stability of shock waves, uniqueness of BV solutions and the inviscid limit from Navier-Stokes equations to small BV solution of Euler equations.

Stability of dispersive shock for the KdV Burgers' equation

Yannan Shen and **Geng Chen**
University of Kansas

In this talk, I will present the recent progress on the L2 stability of dispersive shock for the KdV Burgers' equation. In this project, we will use the method of relative entropy to study the L2 perturbation of dispersive shock, where this idea has been successfully used for both in-viscous and dissipative shock.

Stable singularity formation of the inviscid primitive equations

Quyuan Lin
Clemson University
quyuanl@clemson.edu

Large scale dynamics of the oceans and atmosphere are governed by the primitive equations (PE). While the global well-posedness of viscous PE has been well established, the smooth solutions to the inviscid PE (also known as the hydrostatic Euler equations) can form singularity in finite time. In this paper, I will briefly introduce the inviscid PE, and discuss the stability of a certain type of blowup for smooth solutions.

SESSION 20

Investigations of nonlinear dispersive wave systems

A model for bore propagation with dynamic boundary conditions

Jerry Bona
University of Illinois Chigago
bona@uic.edu

Mathematical models for bore generation and propagation go back to the 1950;s. Most of these models do not allow for cross-river variations. We introduce a model that does make allowance for non-uniform propagation and show that it approximates well solutions of the full, inviscid water-wave model (the Euler equations).

Conditional and unconditional local wellposedness problem for some dispersive systems on the half line

Shenghao Li
School of Mathematical Science, University of Electronic Science and Technology, Chengdu, Sichuan 611731, China
lish@uestc.edu.cn
Xin Yang
School of Mathematics, Southeast University, Nanjing, Jiangsu 211189, China
xinyang@seu.edu.cn
Bingyu Zhang
Department of Mathematics, University of Cincinnati, Cincinnati, OH 45221, USA
zhangb@ucmail.uc.edu

We first consider the conditional local well-posedness problem for initial boundary value problem for dispersive equation via the boundary integral operator method which was developed by

Bona-Sun-Zhang. In this talk, we will use the nonlinear Schrodinger equation and Coupled KdV system as examples to introduce the idea of the approach and the main difficulties under the framework of Bourgain type spaces. Then, due to uniqueness issue raised by the problem, we will introduce a general idea to show the unconditional well-posedness. This is a joint work with Xin Yang and Bingyu Zhang.

1. J. L. Bona, S. M. Sun, and B.-Y. Zhang. A non-homogeneous boundary-value problem for the Korteweg-de Vries equation in a quarter plane. *Transactions of the American Mathematical Society*, 354(2):427–490, 2002.
2. J. L. Bona, S. M. Sun, and B.-Y. Zhang. Boundary smoothing properties of the Korteweg-de Vries equation in a quarter plane and applications. *Dyn. Partial Differ. Equ.*, 3(1):1–69, 2006.
3. M. Cavalcante. The initial boundary value problem for some quadratic nonlinear Schrödinger equations on the half-line. *Differential Integral Equations*, 30(7-8):521–554, 2017.
4. S. Li, M. Chen, X. Yang, and B.-Y. Zhang. Lower regularity solutions of the non-homogeneous boundary-value problem for a higher order Boussinesq equation in a quarter plane. *Nonlinear Anal.*, 221: No. 112893, 29, 2022.

Effect of lower order terms on the well-posedness of Majda-Biello systems

Xin Yang
Southeast University, China
xinyang@seu.edu.cn
Shenghao Li
University of Electronic Science and Technology of China, China
lish@uestc.edu.cn
Bing-Yu Zhang
University of Cincinnati
zhangb@ucmail.uc.edu

This paper investigates a noteworthy phenomenon within the framework of Majda-Biello systems, wherein the inclusion of lower-order terms can enhance the well-posedness of the system. Specifically, we investigate the initial value problem (IVP) of the following system:

$$\begin{gathered} u_t + u_{xxx} = -vv_x, \\ v_t + \alpha v_{xxx} + \beta v_x = -(uv)_x, \\ (u,v)|_{t=0} = (u_0, v_0) \in H^s(\mathbb{R}) \times H^s(\mathbb{R}), \\ x \in \mathbb{R},\ t \in \mathbb{R}, \end{gathered}$$

where $\alpha \in \mathbb{R} \setminus \{0\}$ and $\beta \in \mathbb{R}$. Let $s^*(\alpha,\beta)$ be the smallest value for which the IVP is locally analytically well-posed in $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ when $s > s^*(\alpha,\beta)$. Two interesting facts have already been known in literature: $s^*(\alpha,0) = 0$ for $\alpha \in (0,4) \setminus \{1\}$ and $s^*(4,0) = \frac{3}{4}$. Our key findings include the following:

- For $s^*(4,\beta)$, a significant reduction is observed, reaching $\frac{1}{2}$ for $\beta > 0$ and $\frac{1}{4}$ for $\beta < 0$.
- Conversely, when $\alpha \neq 4$, we demonstrate that the value of β exerts no influence on $s^*(\alpha,\beta)$.

These results shed light on the intriguing behavior of Majda-Biello systems when lower-order terms are introduced and provide valuable insights into the role of α and β in the well-posedness of the system.

Global well-posedness, ill-posedness and long time behavior of solutions of the surface electromigration equation

Deqin Zhou
Chongqing University, China
deqinzhou1952@126.com

We will talk about the Cauchy problem and the long time behavior of the two dimensional surface electro-migration (SEM) equation,which could be regarded as 2D KdV. Firstly, we prove that the SEM equation is locally well-posed in $H^s(\mathbb{R}^2)$ with $s > -\frac{1}{4}$. Secondly, we prove that the SEM equation is ill-posed in $H^s(\mathbb{R}^2)$ with $s < -\frac{1}{4}$ in the sense that the solution map is not C^2. Finally, we get a local energy decay result of the global solution in a growing unbounded in time region but not containing the soliton region. Our well-posedness results extend the latest results in [F. Linares, A. Pastor, M. Scialom, Nonlinearity, 34(2021), 5213–5233], where the SEM equation is proved to be locally well-posed in $H^s(\mathbb{R}^2)$ with $s > \frac{1}{2}$.

On a Schrödinger-Poisson system with doping profile

Mathieu Colin* and **Tatsuya Watanabe****
∗ University of Bordeaux, France, ∗∗ Kyoto Sangyo University, Japan

This talk is devoted to the study of the nonlinear Schrödinger-Poisson system with a doping profile which appears in the study of semi-conductor theory. We are interested in the existence of stable standing waves [1] and make the link with ground states solutions obtained in [2]. The presence of a doping profile causes several difficulties. Especially, in the study of a L^2- minimization constraint problem, the proof of the classical strict sub-additivity is delicate due to the loss of the invariant translation property. This problem is overcommed by adapting a scaling argument inspired by [3].

1. M. Colin, T. Watanabe, Stable standing waves for Nonlinear Schrödinger-Poisson system with a doping profile, *preprint*.
2. M. Colin, T. Watanabe, Ground state solutions for Schrödinger-Poisson system with a doping profile, *preprint*.
3. X. Zhong, W. Zou, A new deduction of the strict sub-additive inequality and its application: ground state normalized solution to Schrödinger equations with potential, *Diff. Int. Eqs.* **36** (2023), 133-160.

On solitary-wave solutions of generalized *abcd*-Boussinesq system with a Hamiltonian structure

Shu-Ming Sun
Virginia Tech
sun@math.vt.edu

The talk discusses the solitary-wave solutions of the so-called *abcd*-Boussinesq system, which is a model of two equations that can describe the propagation of small-amplitude long waves in both directions in water of finite depth. If the system is Hamiltonian, where the parameters b and d satisfy $b = d > 0$, small solutions of the system in the energy space are globally defined. Then, a variational approach is applied to establish the existence and nonlinear stability of the set of solitary-wave solutions for the generalized *abcb*-Boussinesq system. The main point of the analysis is to show that the traveling-wave solutions of the generalized *abcb*-Boussinesq system converge to nontrivial solitary-wave solutions of the generalized Korteweg-de Vries equation. (This is a joint work with R. de A. Capistrano-Filho and J. R. Quintero)

Existence and stability of solitary wave solutions of Boussinesq-Full Dispersion systems for internal waves

A. Durán
University of Valladolid, Spain
angeldm@.uva.es

This talk is concerned with the Boussinesq-Full Dispersion system. This is a three-parameter system of pde's, introduced by Bona, Lannes, and Saut in [1] as a model for the propagation of internal waves along the interface of two-fluid layers with rigid lid condition for the upper layer, and under a Boussinesq regime for the upper layer and a full dispersion regime for the lower layer. Some results on the existence and stability of solitary wave solutions will be discussed.

1. J. L. Bona, D. Lannes, J.-C. Saut, Asymptotic models for internal waves, J. Math. Pues Appl., 89 (2008) 538-566.

Global well-posedness of the initial-boundary value problem of a class generalized KdV equation on a finite interval

Bing-Yu Zhang
University of Cincinnati

In this talk, we consider the initial-boundary value problem (IBVP) of the generalized KdV equation posed on the finite interval $(0,L)$:

$$\begin{cases} u_t + u_x + a(u)u_x + u_{xxx} = 0,\ 0<x<L,\ t>0 u(x,0)=\phi(x) \\ u(0,t)=0,\ u(L,t)=0,\ u_x(L,t)=0 \end{cases}$$

Here, the function $a \equiv a(\mu)$ satisfies the growth condition:

$$a(0)=0,\ |a^{(j)}(\mu)| \le C(1+|\mu|^{p-j}) \quad \forall \mu \in \mathbb{R}$$

for $j=0,1$ if $1 \le p < 2$ and for $j=0,1,2$ if $p \ge 2$.
In the existing literature, the IBVP is known to be globally well-posed in the space $L^2(0,L)$ when $1 \le p < 2$. In this talk we extend these results and show that the IBVP is also globally well-posed in the space $L^2(0,L)$ when $2 \le p < 4$.

Modeling forest ecosystem and comparison principles

Olivier Goubet
Laboratoire Paul Painlevé, CNRS U-Lille et Inria Paradyse

[Canceled]
We introduce some mathematical models for the dynamics of two forest species. We discuss individual models with population models, and how they are related by suitable hydrodynamics limits (large population limits). We use some comparison principles to compute the extinction time of one species.

Control of the intermediate long wave equation on a periodic domain

Lionel Rosier
ULCO, France
Lionel.Rosier@univ-littoral.fr
Francisco Vielma
UTEM, Chile
fvielma@utem.cl

The Intermediate Long Wave (ILW) equation is a mathematical model for the propagation of nonlinear waves at the interface between two fluids of different densities. In this talk, we consider the ILW equation posed on a periodic domain. We show that the ILW equation is both exactly controllable and exponentially stabilizable by using a localized distributed control.

Numerical control of a semilinear wave equation on an interval

M.M. Cavalcanti and **V.N. Domingos Cavalcanti**
Universidade Estadual de Maringá, Brazil
mmcavalcanti@uem, vndcavalcanti@uem.br
C. Rosier and **L. Rosier**
University of Littoral, France
carole.rosier@univ-littoral.fr, lionel.rosier@univ-littoral.fr

We are concerned with the numerical exact controllability of the semilinear wave equation on the interval $(0,1)$. We introduce a Picard iterative scheme yielding a sequence of approximated solutions which converges towards a solution of the null controllability problem, provided that the initial data are small enough. The boundary control, which is applied at the endpoint $x=1$, is taken in the space $H_0^1(0,T)$ for $T=2$. For the linear part, the control input is obtained by imposing a transparent boundary condition at $x=1$. Next, we provide several simulations to show the efficiency of the algorithm, using collocation pseudospectral methods on Chebychev grids to discretize the second order derivative in space in the wave equation.

Korteweg-de Vries-Burgers equation on the half plane

Ivonne Rivas Triviño[1] and **Liliana Esquivel**[2]
[1] Universidad del Valle / Université de Reims, Colombia
[2] Universidad de Puerto Rico recinto Mayagüez, Puerto Rico
ivonne.rivas@correounivalle.edu.co, liliana.esquivel@upr.edu

This talk is focused on analyzing the Korteweg-de Vries-Burgers equation in the negative half-line, $\mathbb{R}^-$, and some results of well-posedness in $H^s(\mathbb{R}^-)$ for $s \ge 0$ and of the Boundary Controllability are shown. New boundary estimates are obtained for the solution of the KdV-B on $\mathbb{R}^-$. Unbounded domains bring challenges for the compactness properties when exact controllability is proved, making it necessary to review some intrinsic properties of each equation.

1. Bona, J., Sun, S., & Zhang, B.-Y. (2008). Non-homogeneous boundary value problems for the Korteweg-de Vries and the Korteweg-de Vries-Burgers equation in a quarter plane. *Ann. I. H. Poincaré-AN*, 25, 1145–1185.
2. Rosier, L. (2000). Exact Boundary Controllability for the Linear Korteweg–de Vries Equation on the Half-Line. *SIAM Journal on Control and Optimization*, 39(2), 331–351.
3. Esquivel L. Rivas I. Well-posedness and Bounded controllability for the Korteweg-de Vries-Burger equation in a half-plane. Submitted.

SESSION 21

Advances in integrable systems and inverse scattering

A numerical Riemann-Hilbert approach to the computation of transform pairs

Kaitlynn Lilly and **Tom Trogdon**
University of Washington
klilly@uw.edu

This research presents a unified methodology integrating spectral theory, Riemann-Hilbert problems, and inverse scattering theory to efficiently derive, and numerically implement, transform pairs associated to time-evolution variable-coefficient partial differential equations (PDEs). More specifically, the approach combines analytical formulae with iterative ODE and Riemann-Hilbert methods to efficiently evaluate the forward and inverse transforms, giving a hybrid analytical-numerical method for such PDEs. The method is demonstrated on transforms arising in the solution of the time-dependent Schrödinger and Dirac equations, producing an accurate and stable time evolution method that does not require time stepping.

Three-way focusing Zakharov-Shabat systems with zero diagonal entry

Francesco Demontis
University of Cagliari
fdemontis@unica.it

In this article we develop the direct and inverse scattering theory of the AKNS system $\mathbf{v}_x = (ik\Sigma + Q(x))\mathbf{v}$, where $\Sigma = \text{diag}(1, 0, -1)$ and $Q(x)$ is an off-diagonal 3×3 $\mathbf{\Sigma}$-focusing potential with entries in $L^1(\mathbb{R})$. We derive the time evolution of the scattering data which, through the inverse scattering transform, lead to the solution of the initial-value problem for a generalized long-wave-short-wave equation. We determine the essential spectrum of the AKNS system rigorously.

This talk is based on a joint work with Cornelis van der Mee (University of Cagliari).

Infinite-order solutions of the AKNS hierarchy and their asymptotic behavior under the mKdV flow

Deniz Bilman
University of Cincinnati
bilman@uc.edu

We show that rogue waves of infinite order emerge in the near-field and large-order limit of a family of simultaneous solutions of the first several (arbitrarily many) flows in the AKNS hierarchy. These solutions are simultaneous solutions of the same flows in a set of rescaled variables and they generalize the rogue waves of infinite order obtained for the focusing nonlinear Schrödinger equation [1, 2, 3]. The mechanism behind the generation of the special simultaneous solution of the hierarchy is the same as for the focusing nonlinear Schrödinger equation: accumulation of infinitely many eigenvalues (counted with multiplicity) of the Zakharov-Shabat problem at the origin in the spectral plane, or equivalently, accumulation of singularities in the associated Riemann-Hilbert problem to form an essential singularity at the origin. We also show that, when restricted to the mKdV flow, the solution exhibits anomalous temporal decay. This is joint work with Elliot Blackstone, Peter Miller, and Giorgio Young.

1. D. Bilman, L. Ling, and P. D. Miller, "Extreme superposition: Rogue waves of infinite order and the Painlevé-III hierarchy," *Duke Math. J.* **169**, 671–760, 2020.
2. D. Bilman and R. Buckingham, "Large-order asymptotics for multiple-pole solitons of the focusing nonlinear Schrödinger equation," *J. Nonlinear Sci.* **29**, 2185–2229, 2019.
3. D. Bilman and P. D. Miller, "Extreme superposition: high-order fundamental rogue waves in the far-field regime," *Memoirs of the AMS* **300**, 2024.

The explicit solution of the Benjamin-Ono equation with general rational (including non-soliton) initial data

Peter D. Miller
University of Michigan
millerpd@umich.edu

We show that the initial-value problem for the Benjamin-Ono equation on $\mathbb{R}$ with $L^2(\mathbb{R})$ rational initial data with only simple poles can be solved in closed form via a determinant formula involving contour integrals. The dimension of the determinant depends on the number of simple poles of the rational initial data only and the matrix elements depend explicitly on the independent variables (t, x) and the dispersion coefficient ϵ. This allows for various interesting asymptotic limits to be resolved quite efficiently. This is joint work with Elliot Blackstone, Louise Gassot, and Patrick Gérard.

Soliton-gas for the focusing nonlinear Schrödinger equation with box-like initial conditions

Aikaterini Gkogkou, Kenneth McLaughlin and **Guido Mazzuca**
Tulane University

This work analyzes the soliton-gas regime of the focusing nonlinear Schrödinger equation with box-like initial conditions. We consider the scenario where the discrete eigenvalues λ_j, $j = 1, 2, \cdots N$, accumulate along a horizontal line in $\mathbb{C}^+$ (while λ_j^* accumulate along the corresponding complex contour in $\mathbb{C}^-$), extending from $-a + ib$ to $a + ib$, where $a, b \in \mathbb{R}_+$. Specifically, the imaginary parts of the eigenvalues are fixed at the value b, while the real parts satisfy the condition $\int_{-a}^{\mathfrak{Re}(\lambda_j)} \rho(x + ib)\, dx = \frac{2j-1}{N}$, where ρ is a density function analytic in a neighborhood of the segment $[-a + ib, a + ib]$. The norming constants are defined by $c_j = h(\lambda_j)$, where h is a function analytic across the same segment. Within this framework, we derive an explicit expression for the solution in the large N limit. It is noteworthy that, in contrast to previous studies that have considered the same regime, either in the mKdV equation [1] or the NLS equation [2], the norming constants do not tend to zero as N increases but instead remain of order $O(1)$.

1. M. Girotti, T. Grava, R. Jenkins, K. T. McLaughlin, and A. Minakov. *Soliton versus the gas: Fredholm determinants, analysis, and the rapid oscillations behind the kinetic equation.* Communications on Pure and Applied Mathematics, 76, p. 3233–3299, (2023).
2. M. Girotti, T. Grava, K. D. McLaughlin, and J. Najnudel. *Law of large numbers and central limit theorem for random sets of solitons of the focusing nonlinear Schrödinger equation.* arXiv preprint arXiv:2411.17036, (2024).

Well-posedness of a higher-order nonlinear Schrödinger equation on a finite interval

Chris Mayo
University of Kansas

The higher-order nonlinear Schrödinger (HNLS) equation is a more accurate alternative to the standard NLS equation when studying wave pulses in the femtosecond regime. It arises in a variety of applications ranging from optics to water waves to plasmas to Bose-Einstein condensates. In this talk, we consider the initial-boundary value problem for HNLS on a finite interval in the case of a power nonlinearity. We establish the local well-posedness of this problem in the sense of Hadamard (existence and uniqueness of the solution as well as its continuous dependence on the data) for initial data in the Sobolev space H^s on a finite interval and boundary data in suitable Sobolev spaces determined by the regularity of the initial data and the HNLS equation.

The proof relies on a combination of estimates for the linear problem and nonlinear estimates, which vary depending on whether $s > 1/2$ or $0 \le s < 1/2$. The linear estimates are established by using the explicit solution formula obtained via the unified transform method of Fokas. This is a joint work with Dionyssis Mantzavinos and Turker Ozsari.

New results in the Riemann-Hilbert problem for the semiclassical defocusing Schrodinger equation

Joanne Dong and **Peter Miller**
University of Michigan
jtdong@umich.edu, millerpd@umich.edu

Solutions to the semiclassical defocusing nonlinear Schrodinger equation with nonzero boundary conditions are studied via the solution to the associated Riemann-Hilbert problem.

Kuznetsov-Ma breather solutions of the defocusing Ablowitz-Ladik equation with large background amplitude

Nicholas Ossi, Evans Boadi and **Barbara Prinari**
University at Buffalo
nossi@buffalo.edu, evansboa@buffalo.edu, bprinari@buffalo.edu

Efstathios Charalampidis
San Diego State University
echaralampidis@sdsu.edu

Panayotis Kevrekidis
University of Massachusetts Amherst
kevrekid@umass.edu

This talk will cover the results of a recent project on the Kuznetsov-Ma (KM) breather solutions of the defocusing Ablowitz-Ladik equation (an integrable spatial discretization of the nonlinear Schrödinger equation) with large background amplitude. These KM solutions are periodic in time and localized in space, and can be obtained via the inverse scattering transform method. While they can generically experience singularity in finite time, conditions under which KM breathers remain regular will be discussed. Additionally, Darboux transformations are employed to construct multi-KM solutions.

A Sasa-Satsuma-mKdV equation and its various soliton solutions

Changyan Shi and **Bao-Feng Feng**
University of Texas Rio Grande Valley
changyan.shi01@utrgv.edu

In this talk, we first propose the Lax pair and its bilinear form of a Sasa-Satsuma-mKdV equation from the ones of the three-component coupled Hirota equation. Then, by the Kadomtsev-Petviashvili (KP) reduction method, we derive its various soliton solutions including the bright, dark, bright-dark, as well as breather solutions. Their dynamical properties are also explored.

Whitham modulation theory for the Davey-Stewartson systems

Alexander Chernyavsky
Texas A&M Corpus Christi
alexandr.chernyavskiy@tamucc.edu

Gino Biondini and **John Ringland**
State University of New York at Buffalo
biondini@buffalo.edu, ringland@buffalo.edu

Whitham modulation theory is developed for the Davey-Stewartson system, a nonlocal extension of the nonlinear Schrödinger equation in 2+1 dimensions. A system of quasilinear first-order PDEs is obtained through a multiple scales expansion and averaging over one oscillation period of the periodic traveling wave solutions. Subsequently, the system is transformed into a hydrodynamic Whitham system, which is then utilized to investigate the transverse stability of periodic traveling wave solutions.

Breather gas fission from elliptic potentials in self-focusing media

Gino Biondini
University at Buffalo, SUNY
biondini@buffalo.edu

Gennady A. El
Northumbria University, UK
gennady.el@northumbria.ac.uk

Xu-Dan Luo
Chinese Academy of Sciences, China
lxdmathematics@gmail.com

Jeffrey Oregero*
University of Kansas
oregero@ku.edu

Alexander Tovbis
University of Central Florida
Alexander.Tovbis@ucf.edu

[Canceled]

We present an analytical model of integrable turbulence for the focusing nonlinear Schrodinger (fNLS) equation, generated by a one-parameter family of finite-band elliptic potentials in the semiclassical limit. We show that the spectrum of these potentials exhibits a thermodynamic band/gap scaling compatible with that of soliton and breather gases depending on the value of the elliptic parameter m of the potential. We then demonstrate that, upon augmenting the potential by a small random noise (which is inevitably present in real physical systems), the solution of the fNLS equation evolves into a fully randomized, spatially homogeneous breather gas, a phenomenon we call breather gas fission. We analytically compute the kurtosis of the breather gas as a function of the elliptic parameter m, and we show that it is greater than two for $0 < m < 1$, implying non-Gaussian statistics. These results establish a link between semiclassical limits of integrable systems and the statistical characterization of their soliton and breather gases.

1. G. Biondini, G. A. El, X-D. Luo, J. Oregero, A. Tovbis, Breather gas fission from elliptic potentials in self-focusing media, Phys. Rev. E **111**, 014204 (2025)

Modeling the downshift of deep water wave trains

C.M. Schober
University of Central Florida

[Canceled]

In this talk we give an overview of two models used to describe the evolution of deep water gravity waves. We investigate a higher-order nonlinear Schrödinger equation with linear damping and weak viscosity (vHONLS), recently introduced as a model for deep water waves exhibiting frequency downshifting. We discuss the impact of viscosity on the linear stability of the Stokes wave solution, its role in enhancing rogue wave formation, and present new criteria for downshifting in the spectral peak. Additionally, we derive a higher order nonlinear Schrödinger equation (NLD-HONLS) which includes damping of the mean flow and propose it as a novel model for frequency downshift.

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2. A. Calini, L. Elisor, C. M. Schober and E.Smith, Studies in Applied Mathematics 152, 513 (2023).

Spectral theory for non-self-adjoint Dirac operators with periodic potentials and inverse scattering transform for the focusing nonlinear Schrödinger equation with periodic boundary conditions

Gino Biondini*, Gregor Kovacic, Alexander Tovbis†, Zachery Wolski** and Zechuan Zhang***
* State University of New York at Buffalo,
** Rensselaer Polytechnic Institute,
† University of Central Florida
zezhang@sissa.it

[Canceled]

We formulate the inverse spectral theory for a non-self-adjoint one-dimensional Dirac operator associated periodic potentials via a Riemann-Hilbert problem approach. We also use the resulting formalism to solve the initial value problem for the focusing nonlinear Schrödinger equation. We establish a uniqueness theorem for the solutions of the Riemann-Hilbert problem, which provides a new method for obtaining the potential from the spectral data. The formalism applies for both finite-genus and infinite-genus potentials. As in the defocusing case, the formalism shows that only a single set of Dirichlet eigenvalues is needed in order to uniquely reconstruct the potential of the Dirac operator and the corresponding solution of the focusing NLS equation.

SESSION 22

Nonlinear waves in lattices

Dispersive shock waves in granular chains

Christopher Chong
Bowdoin College
cchong@bowdoin.edu

Dispersive shock waves (DSWs), which connect states of differing amplitude through a modulated wave train, naturally arise in nonlinear dispersive media experiencing sudden state changes. In this talk, we will examine several DSW approximations in granular chains, including the KdV approximation, the Toda lattice approximation, and various quasi-continuum models. These approximations will be analyzed and compared in detail, with a discussion of their respective strengths and limitations.

Bridging opposing asymptotic descriptions of discrete breathers on nonlinear lattices

Andrew Hofstrand
New York Institute of Technology
ahofstra@nyit.edu

We discuss a ubiquitous class of solutions on nonlinear lattices known as *discrete breathers*, which are periodic in time and localized in space, and bridge opposing asymptotic descriptions of their behavior. In particular, we consider breathers on classical 1d SSH and 2d honeycomb and Kagome lattices, whose geometries commonly appear in physical systems.

On the existence of generalized breathers and transition fronts in time-periodic nonlinear lattices

Dmitry Pelinovsky
McMaster University, Canada
pelinod@mcmaster.ca

We prove the existence of a class of time-localized and space-periodic breathers (called q-gap breathers) in nonlinear lattices with time-periodic coefficients. These q-gap breathers are the counterparts to the classical space-localized and time-periodic breathers found in space-periodic systems. Using normal form transformations, we establish rigorously the existence of such solutions with oscillating tails (in the time domain) that can be made arbitrarily small, but finite. Due to the presence of the oscillating tails, these solutions are coined generalized q-gap breathers. Using a multiple-scale analysis, we also derive a tractable amplitude equation that describes the dynamics of breathers in the limit of small amplitude. In the presence of damping, we demonstrate the existence of transition fronts that connect the trivial state to the time-periodic ones. The analytical results are corroborated by systematic numerical simulations. This is a joint work with Chris Chong and Guido Schneider.

Approximation of Calogero-Moser lattices by Benjamin-Ono equations

J. Doug Wright
Drexel University
jdw66@drexel.edu

We provide a rigorous validation that the infinite Calogero-Moser lattice can be well-approximated by solutions of the Benjamin-Ono equation in a long-wave limit.

Dimer FPUT periodics without symmetry

Timothy Faver
Kennesaw State University
tfaver1@kennesaw.edu

We prove the existence of small-amplitude periodic traveling waves in dimer Fermi-Pasta- Ulam-Tsingou (FPUT) lattices without assumptions of physical symmetry. Such lattices are infinite, one-dimensional chains of coupled harmonic oscillators in which the oscillator masses and/or the coupling potentials can alternate. Previously, periodic traveling waves were constructed in a variety of limiting regimes for the symmetric mass and spring dimers, in which only one kind of material data alternates. The new results discussed here remove the symmetry assumptions by exploiting the gradient structure and translation invariance of the traveling wave problem. Together, these features eliminate certain solvability conditions that symmetry would otherwise manage.

Solitary waves in Fermi-Pasta-Ulam-Tsingou lattices with long range particle interaction

Udoh Akpan
West Texas A&M University
uakpan@wtamu.edu

Doug Wright
Drexel University
jdw66@drexel.edu

We study the existence and properties of certain kind of traveling waves (nearsonic solitary waves) in Fermi-Pasta-Ulam-Tsingou (FPUT) Lattices. These are infinite, one dimensional chain of identical particles arranged on a horizontal line with infinite nonlocal particle neighbors interactions on both sides. We consider both next neighbor particle interactions and long range particle interactions. In both cases, we prove the existence of localized traveling waves which relies on the Implicit Function Theorem. Techniques of Fourier analysis enable us to reformulate the problem to the study of waves that are small perturbations of well known ODEs. Additionally, we deploy the method originally developed by Beale for a capillary water wave problem to prove the existence of nanopterons solutions. The nanopteron wave profiles are formed by the superposition of an exponentially decaying term (a small perturbation of a KdV sech^2 -type soliton) and a periodic term with a very small amplitude. The solution develops a ripple as $|x| \to \infty$. The periodic wave solution is an essential part of our analysis.

Dispersive hydrodynamics for a discrete conservation law

Patrick Sprenger
University of California - Merced
sprenger@ucmerced.edu

The hydrodynamics of a system of conservation laws can often be understood by studying solutions to a Riemann problem, i.e., the evolution of step initial data in a hyperbolic system. When dispersion compensates for the large gradients induced by this initial data, wave-breaking is often resolved by a dispersive shock wave (DSW). An active area of research is the investigation of quantitative features of DSWs in continuum models arising in physical applied mathematics. This talk leverages modern techniques to investigate solutions of the Riemann problem for a semi-discrete conservation law. The semi-discrete model is obtained by applying a first-order centered difference scheme to the spatial derivative of the Hopf equation. Solutions to the Riemann problem reveal a surprisingly elaborate set of solutions to this example system. In addition to discrete analogs of well-known dispersive hydrodynamic solutions—rarefaction waves (RWs) and DSWs—additional unsteady solution families and finite-time blow-up are observed and characterized. We will also compare the dynamics of the Riemann problem to an integrable discretization of the spatial derivative.

SESSION 23

Painlevé equations, integrable systems, and related topics

Tau-functions and monodromy symplectomorphisms

Dmitry Korotkin
Concordia University, Canada
dmitry.korotkin@concordia.ca

We derive a new Hamiltonian formulation of Schlesinger equations in terms of the dynamical r-matrix structure. The corresponding symplectic form is shown to be the pullback, under the monodromy map, of a natural symplectic form on the extended monodromy manifold. We show that Fock-Goncharov coordinates are log-canonical for the symplectic form on the extended monodromy manifold. Using these coordinates we define the symplectic potential on the monodromy manifold and interpret the isomonodromic tau-function as the generating function of the monodromy map. This, in particular, solves a recent conjecture by A.Its, O.Lisovyy and A.Prokhorov. The talk is based on the joint paper with Marco Bertola [Communications in Mathematical Physics **388** 245-290 (2021)].

Asymptotics of Umemura rational Painlevé-V functions

Robert Buckingham
University of Cincinnati
buckinrt@uc.edu

The Painlevé-V equation has two families of rational solutions, one built from generalized Umemura polynomials and one built from generalized Laguerre polynomials. The zeros and poles of both families exhibit remarkable geometric structures. In joint work with Matthew Satter of the University of Michigan, we derive a novel Riemann-Hilbert representation for the rational solutions built from Umemura polynomials and use it to obtain large-degree asymptotic behavior.

Integral transformations of KZ-type equations and the construction of unitary representations of braid groups

Haru Negami
Chiba University, Japan
23wd0102@student.gs.chiba-u.ac.jp

The question of whether unitary representations of braid groups can be constructed through the Long-Moody construction [5] remains an open problem [1]. In this talk, we establish a correspondence between the Katz-Long-Moody construction[3], an algebraic framework for generating infinitely many representations of the braid group B_n, and the multiplicative middle convolution of the Knizhnik-Zamolodchikov (KZ)-type equation[4], a specific integral transformation [2]. Additionally, we demonstrate that the Katz-Long-Moody construction preserves the unitarity of representations, shedding light on the interplay between the Long-Moody construction and the unitarity of these representations.

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Symmetry of q-Painlevé equations and Lax pairs

Kouichi Takemura
Ochanomizu University, Japan
takemura.kouichi@ocha.ac.jp

Discrete Painlevé equations were classified by Sakai [1], and each member of the q-difference Painlevé equation was labelled by the affine Dynkin diagram from the symmetry. It is known that the q-Painlevé equations are expressed by compatibility conditions of Lax pairs [4]. We investigate the symmetry of the linear q-difference equations which are associated with Lax pairs of some q-Painlevé equations [2, 3]. We apply it for adjustment of the expression of the time evolution on the q-Painlevé equations in terms of the Weyl group symmetry.

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Integrable spinning fluid

Andrii Liashyk and **Ivan Sechin**
Beijing Institute of Mathematical Sciences and Applications, China
liashyk@bimsa.cn
Paul Wiegmann
University of Chicago

The one-dimensional Calogero-Moser model is a well-established integrable model describing N interacting particles in both classical and quantum frameworks. In their seminal paper, Abanov, Bettelheim, and Wiegmann [1, 2] demonstrated that a collective description of this model gives rise to integrable hydrodynamics similar to the Benjamin-Ono system.

These interacting particle systems can also be extended to include integrable spin generalizations, where internal degrees of freedom are assigned to each particle. In our paper we discuss the hydrodynamic formulation of the spin Calogero-Moser system, which emerges as an integrable spinning fluid system, and elucidate its connection to the matrix Benjamin-Ono equation recently introduceed in [3].

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Affine Weyl group actions on a 3×3 Lax form for the q-$E_6^{(1)}$ Painlevé equation

Kanam Park
Toba College, Japan
paku-k@toba-cmt.ac.jp

The q-Painlevé equation of type $E_6^{(1)}$ was given as a q-difference system with an affine Weyl group symmetry of type $E_6^{(1)}$. In the previous work [1], we gave a new 3×3 matrix Lax form for the equation. On the Lax form, there were bi-rational Weyl group actions [2] which generate the $W(A_2 \times A_2 \times A_2)$.

Our goal is to understand affine Weyl group actions on the q-$E_6^{(1)}$ equation as actions on the Lax form. In this paper, we show the Weyl group actions [2] and a correspondence between them and affine Weyl group actions on the q-$E_6^{(1)}$ equation.

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2. Park, K. and Yamada, Y., "Symmetry of factorized Lax matrices", RIMS Kôkyûroku Bessatsu, **B 87**, (2021), 133–145.

Real and imaginary roots of generalised Okamoto polynomials and partial-rogue waves in the Sasa-Satsuma equation

Alexander Stokes
Waseda University, Japan
stokes@aoni.waseda.jp
Pieter Roffelsen
University of Sydney, Australia
pieter.roffelsen@sydney.edu.au

While solutions of the fourth Painlevé equation are in general transcendental, for special parameter values there are rational solutions, including a hierarchy of those expressed in terms of generalised Okamoto polynomials [1]. Recently, B. Yang and J. Yang [2] derived a family of rational solutions to the Sasa-Satsuma equation and showed that any of its members constitutes a “partial-rogue wave" provided that an associated generalised Okamoto polynomial has no real roots or no imaginary roots.

Roots of these polynomials correspond to singularities of the rational solutions, and we develop an algorithmic procedure to derive the qualitative distribution of singularities on the real line for real solutions of Painlevé equations generated by Bäcklund transformations, starting from the known distribution for a seed solution. This procedure is based on the Okamoto-Sakai theory of complex algebraic surfaces associated with Painlevé equations, and in particular the action of Bäcklund transformations on the surfaces and the paths parametrised by real solutions in the real slice of Okamoto's space. We apply this procedure to the rational solutions expressed in terms of generalised Okamoto polynomials and as a corollary obtain exact formulas for the number of real and the number of imaginary roots.

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SESSION 24

Recent advances in photonic systems

tems whose edges are surrounded by air, as well as photonic crystal slabs whose surfaces radiate into the surrounding free space environment. Finally, I will show how to extend this approach to nonlinear systems, to develop both a quantitative definition of topological nonlinear modes and topological dynamics [2].

1. K. Y. Dixon, T. A. Loring, and A. Cerjan, Classifying Topology in Photonic Heterostructures with Gapless Environments, *Phys. Rev. Lett*, **131** (2023), 213801.
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Pure-quartic solitons with PT-symmetric nonlinearity

Savvas Sardelis and **Ziad Musslimani**
Florida State University
ssardelis@fsu.edu, zmuslimani@fsu.edu
Andrea Blanco-Redondo
University of Central Florida
andrea@creol.ucf.edu

The idea of having solitary waves in Kerr nonlinear media arising in the presence of only quartic dispersion was briefly theoretically considered in the early 90's [1] and then almost forgotten until its experimental discovery in 2016 [2]. These so-called pure-quartic solitons (PQS) were observed in a silicon photonic crystal waveguide where quartic dispersion was the dominant dispersion effect and all the other dispersion orders were negligible.

We propose a new class of soliton based on the interaction of parity-time (PT) symmetric nonlinearity and quartic dispersion or diffraction. This novel kind of soliton is related to the recently discovered pure-quartic solitons, that arise from the balance of Kerr nonlinearity and quartic dispersion, through a complex coordinate shift. We find that the PT-symmetric pure-quartic soliton presents important differences with respect to its Hermitian (Kerr) counterpart, including a nontrivial phase structure, a skewed spectral intensity, and a higher power for the same propagation constant. Further analysis reveals these solitons are linearly stable.

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Local topological classification of open and nonlinear photonic systems

Alexander Cerjan
Sandia National Laboratories
awcerja@sandia.gov

Photonic topological insulators exhibit bulk-boundary correspondence, which requires that boundary-localized states appear at the interface formed between topologically distinct insulating materials. However, many topological photonic devices share a boundary with free space, either on their sides or their surfaces, which raises a subtle but critical problem as free space is gapless for photons above the light line. Thus, traditional definitions of topological invariants cannot be rigorously applied – in the absence of a global bulk bandgap common to all of the materials in the system, there is no clear definition of topological protection, nor can one calculate invariants based on integrating over completely occupied bands. Here, we use a local theory of topological materials to resolve bulk-boundary correspondence in heterostructures containing gapless materials and in radiative environments [1]. In particular, we construct the heterostructure's spectral localizer, a composite operator based on the system's real-space description that provides a local marker for the system's topology and a corresponding local measure of its topological protection; both quantities are independent of the material's bulk band gap (or lack thereof). Using this framework, we demonstrate how to rigorously classify topology both in photonic sys-

Edge modes in topological photonic crystals

Junshan Lin
Auburn University
jzl0097@auburn.edu
Wei Li
DePaul University
wei.li@depaul.edu
Jiayu Qiu and **Hai Zhang**
Hong Kong University of Science and Technology, China
haizhang@ust.hk

In this talk, I will present mathematical theory for the existence of edge modes in several topological photonic crystals. Special focus will be on the periodic layered media (1D problem) and the 2D honeycomb lattice. For the former, the edge mode is investigated by using the transfer matrix, while the latter, we develop layer potential method to study the edge mode bifurcated from the Dirac point.

1. Junshan Lin and Hai Zhang, Mathematical theory for topological photonic materials in one dimension, *J. Phys. A. - Math. Theor.*, **55** (2022), 495203.
2. Wei Li, Junshan Lin, Jiayu Qiu and Hai Zhang, Mathematical Theory for Honeycomb Topological Photonic Crystals with Broken Reflection Symmetry, submitted.

Modeling 1D topological insulators in the presence of noisy data

Michael Nameika and **Justin Cole**
University of Colorado Colorado Springs
mnameika@uccs.edu, jcole13@uccs.edu

When measuring topological insulators in a lab, one can reasonably expect to have noisy or disordered data. We have implemented a nonlinear least squares algorithm to derive and compute interaction coefficients for an SSH-type tight binding model for the linear one-dimensional Schrodinger equation for topological insulator systems. We inspect the performance of this algorithm when presented with disordered data, its ability to reproduce the topological properties of the underlying data, and convergence under many realizations.

Bulk and edge solitons of the nonlinear Haldane model

Troy Johnson
UCCS
tjohnso3@uccs.edu

There has been a substantial amount of research devoted to linear topological insulators, but not nearly as much for nonlinear systems. A nonlinear extension of the well-known Haldane model is proposed and studied. Several families of bulk and edge solitons in the nonlinear Haldane model are computed and analyzed. Important properties of the solitons, such as topology, mobility, and stability will be discussed. The similarities and differences between the linear and nonlinear problems will be discussed.

Exploring new frontiers in branched flow of light

Miguel A. Bandres, Bradford Geiger, Enmanuel Guzman and **Konrad Tscherning**
University of Central Florida
bandres@creol.ucf.edu

When waves propagate through a homogeneous medium, they diffract and spread. However, when waves propagate through a smooth disordered medium, instead of producing completely random patterns, they form channels of enhanced intensity that keep dividing as they propagate, forming a beautiful pattern resembling the branches of a tree. This fundamental wave phenomenon is known as branched flow and has often been overlooked. It was first observed for electrons, but it is a universal wave phenomenon that can occur for any kind of wave, from microwaves to sound and tsunami waves. Recently, we observed the branched flow of light by studying light propagation in thin liquid films [1]. This new experimental platform not only enables a deeper exploration of the key characteristics of branched flow but also allows us to experimentally verify theoretically predicted scaling laws governing its universal characteristic lengths. Furthermore, this photonic platform opens new avenues for studying branched flow in previously unexplored regimes, including the effects of optical and thermal nonlinearities as well as optofluidic interactions. In this talk, we will present entirely new aspects of the branched flow of light that we have recently observed.

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SESSION 25

Emergent dynamics in integrable systems: soliton gasses and related topics

Soliton train

G. Mazzuca
Tulane University
gmazzuca@tulane.edu

We investigate the focusing nonlinear Schrödinger equation (fNLS) in the solitonic regime, introducing randomness into the eigenvalues of the scattering data. Specifically, we consider N eigenvalues of the form $z_k = (-\tau_k + i\mu_k)/2$, $k = 1, \ldots, N$ where the μ_k are i.i.d. sub-exponential random variables and τ_k are deterministic such that $\tau_{k+1} - \tau_k > \Delta(N)$. We derive explicit deterministic and probabilistic results, such as the law of large numbers and a central limit theorem (CLT) for the solution's profile.

Our analysis reveals that at a space-time collision point, the solution exhibits a universal behavior: it converges to a scaled $\text{sinc}(x) = \frac{\sin(x)}{x}$ profile. Additionally, we show that the constructive interference of N solitons yield a peak amplitude of order N, highlighting an alternative mechanism for rogue wave formation. The talk is based on an ongoing project with Manuela Girotti (Emory University), Tamara Grava (SISSA & University of Bristol), Robert Jenkins (University of Central Florida), Ken McLaughlin (Tulane University) and Maxim Yattselev (Indiana University Indianapolis).

Planar orthogonal polynomials with non-Hele-Shaw type polynomial potentials

Seung-Yeop Lee
University of South Florida
lees3@usf.edu

Abril Arenas
University of South Florida
louisarenas@usf.edu

Planar orthogonal polynomials in the double scaling limit have been much studied for their connection to Coulomb gas system in two dimensions. Most exact results have been known either for radially symmetric potential or for so-called Hele-Shaw potential, where the limiting density of the Coulomb gas is uniform over its support. When the potential is not Hele-Shaw type nor radially symmetric, we expect to observe a new type of singular behaviors. In such cases there is no multiple orthogonality that we can use for asymptotic analysis of the planar polynomials.

In this talk, we will propose a matrix Riemann-Hilbert problem for some polynomial potential that is not radially symmetric and not Hele-Shaw type. More explicitly we will consider the case when the Laplacian of the potential is $|z|^{2p}$ for any positive integer p. This work is a preliminary report of the work by Abril Arenas.

Focusing NLS condensates of minimal averaged intensity

Alexander Tovbis
University of Central Florida
Alexander.Tovbis@ucf.edu

Marco Bertola
Concordia University
marco.bertola@concordia.ca

We give a nutshell introduction of spectral theory of soliton gases for integrable equations and then discuss soliton condensates for the focusing Nonlinear Schrödinger (fNLS) equation, which are determined by their Schwarz symmetric spectral support sets in $\mathbb{C}$. We consider the problem of minimizing the average intensity of an fNLS condensate whose support is a continuum (or a polycontinuum) containing a given "anchor set" of finitely many points in the upper half plane.

Wave-mean field interaction in integrable turbulence

Thibault Congy and **Gennady A. El**
Northumbria University, UK
thibault.congy@northumbria.ac.uk, gennady.el@northumbria.ac.uk

Mark A. Hoefer
University of Colorado Boulder
hoefer@colorado.edu

Motivated by previous work on the interaction of individual solitons with slowly varying or multiscale mean fields [1], an analytical model for wave-mean field interaction in integrable turbulence [2] is developed. The model results from considering the KdV soliton gas kinetic equation [3] with variable spectral support. The introduction of a composite ansatz for the soliton gas density of states explicitly identifies a genuinely nonlinear mean field related to the so-called soliton condensate [4], an induced mean term, and a "dressed" soliton gas. As an application, a polychromatic soliton gas [5] interaction with mean fields initiated by initial steps for the KdV equation (rarefaction and dispersive shock waves) is studied.

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Universality in the small-dispersion limit of the Benjamin-Ono equation

Elliot Blackstone, Peter Miller and **Matthew Mitchell**
University of Michigan
mattdmit@umich.edu

We examine the solution of the Benjamin-Ono Cauchy problem for rational initial data in three types of double-scaling limits; the dispersion is taken to zero while simultaneously the independent variables either approach a point on one of the two branches of the caustic curve of the inviscid Burgers equation, or approach the critical point where the branches meet. Our method involves steepest descent analysis on contour integrals appearing in an explicit representation of the solution of the Cauchy problem. Our results reveal universal limiting profiles in each case that are independent of details of the initial data.

Non-standard Green energy problems in the complex plane

Abey López-García and **Alexander Tovbis**
University of Central Florida
abey.lopez-garcia@ucf.edu, alexander.tovbis@ucf.edu

Motivated by the study of the existence and uniqueness of solutions of the nonlinear dispersive equations in the theory of soliton gases analyzed in [1], we consider several non-standard discrete and continuous Green energy problems in the complex plane and study the asymptotic relations between their solutions. In the discrete setting, we consider two problems; one with variable particle positions (within a given compact set) and variable particle masses, the other one with variable masses but prescribed positions. The mass of a particle is allowed to take any value in the range $0 \le m \le R$, where $R > 0$ is a fixed parameter in the problem. The corresponding continuous energy problems are defined on the space of positive measures μ with mass $\|\mu\| \le R$ and supported on the given compact set, with an additional upper constraint that appears as a consequence of the prescribed positions condition. It is proved that the equilibrium constant and equilibrium measure vary continuously as functions of the parameter R (the latter in the weak-star topology). In the unconstrained energy problem we present a greedy algorithm that converges to the equilibrium constant and equilibrium measure. In the discrete energy problems, it is shown that under certain conditions, the optimal values of the particle masses are uniquely determined by the optimal positions or prescribed positions of the particles, depending on the type of problem considered. This talk reports on results obtained in [2].

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2. A. López-García and A. Tovbis, Non-standard Green energy problems in the complex plane, *Anal. Math. Phys.*, **13** (2023), 77.

The small dispersion limit of KdV

Kurt Schmidt, Matt Mitchell and **Robert Buckingham**
University of Cincinnati
schmidku@mail.uc.edu

We study the small-dispersion KdV equation

$$u_t + 6uu_x + \varepsilon^2 u_{xx} = 0$$

with initial data $u_0(x) = \operatorname{sech}^2(x)$. When $\varepsilon = \varepsilon_N = (N(N+1))^{-1/2}$, the solution takes the form of the Kay-Moses solution, known as the N-soliton. These solutions are numerically difficult to compute for large N. In this talk, we analyze the asymptotic behavior as $N \to \infty$ using the Deift-Zhou nonlinear steepest descent method for Riemann-Hilbert problems.

SESSION 26

Waves in complex mathematical models of optics and superconductivity

Kink dynamics in modified sine-Gordon model

Jacek Gatlik
AGH University of Krakow, Poland
jacek.gatlik@agh.edu.pl

Tomasz Dobrowolski
University of the National Education Commission in Krakow, Poland

Panayotis G. Kevrekidis
University of Massachusetts

Our research focuses on an extensive analysis of the dynamics of kink solutions in a modified sine-Gordon model, including a comprehensive study of the effects of breaking translational invariance due to the presence of periodic and localized inhomogeneities.

We present a significant advancement in understanding the behavior of kink solutions within the sine-Gordon model, particularly in these complex environments. By introducing a novel ansatz, we have successfully constructed an effective model with two degrees of freedom. This model achieves remarkable accuracy in predicting the kink's dynamics, even in the non-perturbative regime and at relativistic velocities. The effectiveness of our model was validated through comprehensive numerical simulations, which demonstrated excellent agreement with the original partial differential equations. We also emphasize the critical influence of initial conditions on the accuracy of the effective model, offering new insights into the interaction between kinks and heterogeneous environments.

Furthermore, we explore the dynamics of kinks under additional influences, such as a switched bias current and dissipation, within environments featuring periodic heterogeneity. Our findings suggest that the effective model not only captures the kink's position and width evolution with high precision but also offers potential for manipulating the kink's trajectory and rest position through external controls. This capability could lead to innovative techniques for controlling nonlinear wave dynamics in various physical systems. The successful application of this approach to more complex environments underscores its robustness and opens new avenues for exploring kink dynamics in other non-integrable Klein-Gordon models and beyond.

Mathematical models of flux trapping by superconductors

Jean-Guy Caputo
INSA Rouen Normandie, France
jean-guy.caputo@insa-rouen.fr

One hundred years ago Meissner discovered that, in the presence of a magnetic field, superconductors create a current that induces a magnetic field directly opposite the applied field. This effect is now being used with high temperature superconductors to levitate trains (maglev), to screen magnetic fields in medical instruments and in many other applications. It is then particularly important to understand how such a material reacts to an external magnetic field.

The basic unit of magnetic flux is the magnetic vortex, it has a typical size $10^{-6}m$. These vortices are described by the Ginzburg-Landau equations for the vector potential and the superconducting density. I will show how Maxwell's equations can be coupled with these equations leading to a Maxwell-Ginzburg-Landau model. In particular we considered how a magnetic pulse scatters on a superconducting layer inducing vortices inside it.

The above description is correct for small samples ($10^{-4}m$). When the sample is larger, there are too many vortices and a macroscopic approach is necessary. It involves Maxwell's equations and a phenomenological constitutive equation $E(J)$ relating the current density J to the electric field E. This equation involves the critical current density J_c which depends on the magnetic field B and the temperature T. We will analyze mathematically this model which consists in a nonlinear convection-diffusion equation for B coupled to a diffusion equation for T.

On the dynamics in a nonlocal perturbed Ablowitz-Ladik lattice

Austin Marstaller and **Alejandro Aceves**
Southern Methodist University
AMarstaller@smu.edu

There is a long and rich story of nonlinear dynamics in discrete lattices. A particular well studied model is the discrete Nonlinear Schrödinger Equation, with many applications, most notably in nonlinear optics.

Recent studies have explored dynamics in scenarios where coupling amongst the array elements is nonlocal. The interest in particulr is that in the longwave approximation, it leads to fractional diffraction. In this contribution, we consider such nonlocal coupling as a perturbation of the integrable Ablowitz Ladik model. By use of perturbation methods, asymptotics and numerical simulations, we present results on modulational instabilities, the emergence of coherent structures and mobility properties of localied modes.

SESSION 27

Recent theoretical and numerical advances in fluids and nonlinear optics

Topological insulators in magneto-optical systems

Justin Cole
University of Colorado, Colorado Springs
jcole13@uccs.edu

Applying an external magnetic field to a 2D array of ferrite rods induces topologically protected chiral states. A tight-binding model to describe this system is developed. The effective discrete model is found to be a generalization of the well-known Haldane model. Properties such as topological invariants and solitons are explored. Connections to Floquet topological insulators and Dirac systems will be highlighted.

Turbulence via intermolecular potential

Rafail V. Abramov
University of Illinois at Chicago
abramov@uic.edu

I will present a novel theory of spontaneous development of turbulence in gases at low Mach numbers, which is consistent with observations. According to my theory, at low Mach numbers, the pressure variable is damped via the imbalance between the incident and recedent pairs of molecules in the pair correlation function of the Vlasov collision integral. As a result, the van der Waals effect becomes non-negligible under the pressure gradient in the momentum transport equation, and couples the density and the divergence of velocity into a linearly unstable, chaotic, rapidly oscillating system. In turn, this chaotic density-divergence system produces turbulent dynamics by exponentially amplifying small noisy fluctuations around a laminar flow. I will also show animations of a numerically simulated turbulent flow, which look visually similar to observations and experiments.

Local and global well-posedness for the Maxwell-Bloch equations with inhomogeneous broadening

Barbara Prinari
University at Buffalo
bprinari@buffalo.edu

The Maxwell-Bloch system of equations with inhomogeneous broadening is studied, and the local and global well-posedness of the corresponding initial-boundary value problem is established by taking advantage of the integrability of the system and making use of the corresponding inverse scattering transform. A key ingredient in the analysis is the L2-Sobolev bijectivity of the direct and inverse scattering transform established by Xin Zhou for the focusing Zakharov-Shabat problem.

Generalized rarefactions and dispersive shock waves for the

Korteweg-de Vries Equation

Yifeng Mao
University of California, San Diego
yim024@ucsd.edu

Gino Biondini
State University of New York, Buffalo

Gennady El
Northumbria University, UK

Mark Hoefer
University of Colorado Boulder

Nonlinear wave interactions ubiquitously arise and lead to rich phenomena in dispersive hydrodynamics. A primary scenario is the Riemann problem, which examines the evolution of discontinuous, step-like initial data between two constant states, forming structures such as rarefaction waves (RWs) and dispersive shock waves (DSWs) in dispersive media. This work extends the Riemann problem to address interactions between two periodic traveling waves, termed the generalized Riemann problem, for the Korteweg–De Vries equation. The examination starts with special classes of two-phase wave interactions consisting of two cnoidal wave solutions subject to a rapid transition in one wave parameter, yielding the long-term emergence of two-phase *bright/dark breather shock waves (BSWs)* characterized by a bright/dark breather train edge and a harmonic edge, as well as one-phase generalized RWs with internal oscillations. The characteristic velocities of the waves are compared with predictions from multiphase Whitham modulation theory. These special cases serve as building blocks for the comprehensive characterization of the generalized Riemann problem.

A Hamiltonian Dysthe equation for water waves

Adilbek Kairzhan
Nazarbayev University, Kazakhstan
akairzhan@nu.edu.kz

In this talk, we present the derivation of a Hamiltonian Dysthe equation for the slowly varying envelope of modulated wavetrains based on a Hamiltonian formulation of the water wave problem and by applying techniques from Hamiltonian perturbation theory. The models we consider include surface gravity waves, water waves with constant vorticity and hydroelastic waves. The latter model is further complicated by the presence of cubic resonances for which a detailed analysis is given.

We also provide the observations on how various physical parameters affect the modulational instability of Stokes waves.

The talk is based on a series of works with Catherine Sulem (University of Toronto) and Philippe Guyenne (University of Delaware) [1, 2, 3].

1. P. Guyenne, A. Kairzhan, C. Sulem. A Hamiltonian Dysthe equation for hydroelastic waves in a compressed ice sheet. *Preprint: arXiv:2410.05630* (2024).
2. Guyenne P., Kairzhan A. and Sulem C. Hamiltonian Dysthe Equation for Three-Dimensional Deep-Water Gravity Waves. *SIAM: Mult. Model. Simul.* **20, no. 1** (2022) pp. 349-378.
3. Guyenne P., Kairzhan A. and Sulem C. A Hamiltonian Dysthe equation for deep-water gravity waves with constant vorticity. *J. Fluid Mech.* **949** (2022) A50.

Exact solution and integrability of ballistic motion of fluid with free surface

Pavel M. Lushnikov
University of New Mexico
plushnik[at]math.unm.edu

A fully nonlinear dynamics for potential flow of ideal incompressible fluid with a free surface is considered in two dimensional geometry. A fluid is assumed to be moving at large distances either towards or away from the origin which can be considered as a version of ballistic motion. An infinite set of exact solution is found including formation of droplets and cusps on a free surface. These solutions are characterized by motion of complex singularities outside of fluid and can be obtained from fully integrable exact reductions of fluid dynamics.

Computing nearly-extreme Stokes waves with high-precision using conformal maps

Denis Silantyev
University of Colorado at Colorado Springs
dsilanty@uccs.edu

Two-dimensional potential flow of the ideal incompressible fluid with free surface and infinite depth has a class of solutions called Stokes waves which is fully nonlinear periodic gravity waves propagating with the constant velocity. We developed a new highly efficient method for computation of nearly-extreme Stokes waves. The convergence rate of the numerical approximation by a Fourier series is determined by the complex singularity of the travelling wave in the complex plane above the free surface. We study this singularity and use an auxiliary conformal mapping which moves it away from the free surface thus dramatically speeding up Fourier series convergence of the solution. Three options for the auxiliary conformal map are described with their advantages and disadvantages for numerics. Their efficiency is demonstrated for computing Stokes waves near the limiting Stokes wave (the wave of the greatest height) with 100-digit precision. Drastically improved convergence rate significantly expands the family of numerically accessible solutions and allows us to study the oscillatory approach of these solutions to the limiting wave at a level of detail previously not available.

CONTRIBUTED PAPERS

Stabilization and SMC design for coupled ODE-transport PDE systems

G. Sangeetha
K. Mathiyalagan
KGISL Institute of Technology, India
kmathimath@gmail.com
S.M. Lee
Kyungpook National University, Korea
moony@knu.ac.kr
Ju H. Park
Yeungnam University, Korea
jessie@ynu.ac.kr

This paper investigates the boundary control problem for first-order ODE-transport PDE coupled at a boundary point. The sliding mode control and linear quadratic regulator techniques are used to design the desired boundary control input. An integral switching surface is introduced to obtain the desired characteristics for the proposed model. Using the Lyapunov approach and linear matrix inequality technique, a sufficient condition for stability is derived. Further, an effective control law is designed to ensure the finite-time convergence of the system. Finally, to illustrate the significance and efficiency of the proposed control design, a numerical example is provided.

1. S.D. Mulje and R.M. Nagarale, LQR technique based second-order sliding mode control for linear uncertain systems, *Int. J. Comput. Appl.*, **137**, (2016) 23-29.
2. J.J. Gu and J.M. Wang, Sliding mode control for N-coupled reaction diffusion PDEs with boundary input disturbances, *Int. J. Robust. Nonlin*, **29** (2019), 1437–1461.
3. W. Kang and E. Fridman, Sliding mode control of schrodinger equation-ODE in the presence of unmatched disturbances, *Syst. Contr. Lett*, **98**, (2016), 65–73.

Vacuum dambreak problem for the nonlinear Schrödinger equation in a harmonic potential

Shashwat Sharan, Patrick Sprenger, Boaz Ilan
UC Merced
ssharan2@ucmerced.edu, sprenger@ucmerced.edu, bilan@ucmerced.edu
Mark Hoefer
CU Boulder
hoefer@colorado.edu

Confined Bose-Einstein condensates (BECs) can be modeled by the Gross-Pitaevskii (GP) / nonlinear Schrödinger (NLS) equation with a harmonic potential. In a vacuum dambreak scenario, the system begins with an initial jump in density which evolves into a rarefaction wave (RW) influenced by the potential. The defocusing 1D NLS without a potential admits simple self-similar RW solutions, characterized by one of the dispersionless Riemann invariants (RI) being constant. Such simple wave solutions are well-studied in the context of Riemann problems. We study the vacuum dam-break problem with a harmonic potential, where simple waves do not exist. We find exact closed-form solutions for the 1D NLS equation with a harmonic potential for such "harmonic rarefaction waves" which preserve self-similarity of the second kind and reduce to simple RW solutions in the zero-potential limit. These solutions are used to construct solutions to the Riemann vacuum dambreak problem in a harmonic potential. Additionally, we also construct approximate solutions for the ground-state initial data and compare the 1D results with full 3D numerical simulations of the NLS equation for experimentally relevant parameters.

Artificial intelligence with Levenberg-Marquardt for calculating the thermal performance of convective radiative straight fin with the triangular shape

Iftikhar Ahmad, Hina Zahid
University of Gujrat, Pakistan
dr.iftikhar@uog.edu.pk, hina.qureshi@uog.edu.pk
Hira Ilyas
University of Chenab, Pakistan
hira@phs.uchenab.edu.pk
Muhammad Asif Zahoor Raja
National Yunlin University of Science and Technology, Taiwan
rajamaz@yuntech.edu.tw
Amna Munir
University of Gujrat, Pakistan
19011709-014@uog.edu.pk

This paper study the distribution of temperature in convective radiative straight type fins with the instantaneous variational thermal conductivity, coefficient of heat transfer, surface emissivity and internal heat generation using temperature. During the measurement of triangular shape of longitudinal fins, convection and radiation sink temperatures have been assumed to be non-zero through out the study. We used the shooting method for the generation of the datasets to supervise the computational learning scheme and trained the datasets with Levenberg-Marquardt artificial neural networks algorithm (LBMANN) to check the effect of different parameters on the triangular shape of the fin.

Mathematical modeling and analysis of leptospirosis transmission dynamics in human and rodent populations with direct transmission

Alishba Sana[1], Adnan Aslam[1], Takasar Hussain[2], Muhammad Ozair[2]
[1]: National University of Sciences and Technology, Pakistan
[2]: COMSATS University Islamabad, Pakistan
adnan.aslam@seecs.edu.pk, alishba.msmath23sns@student.nust.edu.pk, htakasarnust@gmail.com, researchozair@gmail.com

This study aims to develop and analyze a compartmental mathematical model to explore the transmission dynamics of rodent-borne leptospirosis in human populations, incorporating direct transmission, environmental pathogen load, and the interaction between infected rodents and the environment. The analysis begins with an examination of the model's fundamental properties, equilibrium points, and their stability. The basic reproduction number (R_0) is derived using the next-generation matrix approach, indicating that the disease-free equilibrium (DFE) is globally asymptotically stable if $R_0 < 1$ and unstable otherwise. The local stability of the endemic equilibrium is analyzed through center manifold theory, revealing the occurrence of forward bifurcation. Sensitivity analysis is conducted to identify the most influential parameters on the disease dynamics using the normalized for-

ward sensitivity index. Numerical simulations are performed using the fourth-order Runge-Kutta method to illustrate the stability behavior of the endemic equilibrium and the effects of key parameters, including human transmission rates, human recovery rates, and rodent mortality rates, on disease dynamics. Simulation results indicate that when $R_0 > 1$ all trajectories of the model converge to a unique endemic equilibrium over time. Furthermore, the findings emphasize that reducing transmission rates, increasing recovery rates, and implementing strategies to control the rodent population significantly mitigate the spread of the disease within the population. The results are presented graphically to provide a comprehensive understanding of the model's dynamics.

Water waves with vorticity and the Schwarz function

Darren Crowdy
Imperial College London
d.crowdy@imperial.ac.uk

The theory of water waves is centuries old, but it remains a vibrant area of research. Most theoretical work on water waves takes the flow to be irrotational, but there is growing recent interest in the effect of vorticity on the structure of the waves. The assumption of irrotationality has the theoretical advantage that complex analysis techniques can be used to analyze the problem in the two-dimensional setting. This talk will present a novel theoretical formulation of the problem of steadily-travelling water waves in the presence of vorticity (where the assumption of irrotationality is dropped). The approach is based on the notion of a Schwarz function of a curve. It unifies our understanding of several recent results in the water wave literature and provides a wealth of new exact mathematical solutions to this challenging free boundary problem.

Numerical analysis on neural network projected schemes for approximating one dimensional wasserstein gradient flows

Wuchen Li
University of South Carolina
wuchen@mailbox.sc.edu

We provide a numerical analysis and computation of neural network projected schemes for approximating one-dimensional Wasserstein gradient flows. We approximate the Lagrangian mapping functions of gradient flows by the class of two-layer neural network functions with ReLU (rectified linear unit) activation functions. The numerical scheme is based on a projected gradient method, namely the Wasserstein natural gradient, where the projection is constructed from the L2 mapping spaces onto the neural network parameterized mapping space. We establish theoretical guarantees for the performance of the neural projected dynamics. We derive a closed-form update for the scheme with well-posedness and explicit consistency guarantee for a particular choice of network structure. General truncation error analysis is also established on the basis of the projective nature of the dynamics. Numerical examples, including gradient drift Fokker-Planck equations, porous medium equations, and Keller-Segel models, verify the accuracy and effectiveness of the proposed neural projected algorithm. This is based on a joint work with Xinzhe Zuo (UCLA), Jiaxi Zhao (NUS), Shu Liu (UCLA), and Stanley Osher (UCLA).

The initial-value problem for a Gardner-type equation

Jerry L. Bona
University of Illinois Chicago
jbona@uic.edu
Hongqiu Chen
University of Memphis
hchen1@memphis.edu
Pamela Guerrero
University of Memphis
pgrrero3@memphis.edu
Cristina A. Haidau
Lake Forest College
chaidau@lakeforest.edu, cahaidau@neiu.edu
Sanja Pantic
Virginia Polytechnic Institute
sanja@vt.edu, spantic@radford.edu

The Gardner equation

$$u_t + u_x + uu_x + Au^2u_x + u_{xxx} = 0, \qquad (1)$$

where $A \neq 0$ is a constant and $u = u(x,t)$ is a real-valued function of $x \in \mathbf{R}$ and $t \geq 0$ resulted from a Galilean transformation being applied to the modified Korteweg-de Vries equation. One of the many amazing aspects of the Korteweg-de Vries equation itself falls to Gardner's analysis, namely the existence of infinitely many conservation laws in the form of polynomials in the solution and its spatial derivatives [1].

It is our purpose in this paper to introduce a so-called regularized version of the Gardner equation, namely

$$u_t + u_x + uu_x + Au^2u_x - u_{xxt} = 0, \qquad (2)$$

where A is a constant that remains after u, x and t have all been scaled to present the tidy form above that arises in hydrodynamics and plasma physics. Considered here are well-posedness issues for the initial-value problem for the regularized equation posed on all of $\mathbf{R}$.

1. R. M. Miura, *The Korteweg-de Vries Equation: A Survey of Results*, SIAM Rev. **18** (1976) 412-459.

Stability and instability of bound states for a regularized NLS equation

John Albert and **Jack Arbunich**
University of Oklahoma
jalbert@ou.edu, jack.arbunich@ou.edu

We investigate the stability and instability of bound-state solutions to a regularized nonlinear Schrodinger equation derived by Dumas, Lannes, and Szeftel as a model for the propagation of laser beams. The effect of the regularization is to extend the range of nonlinearities for which bound states are stable.